Phenomenology of Transverse-Spin and Transverse-Momentum Effects in Hard Processes

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Transverse spin effects at high energies are often suppressed (e.g., in DIS), but ... There are high-energy hadronic processes where this is not true, and transverse polarization gives a leading contribution: e.g., transversely polarized Drell–Yan production [Ralston and Soper 1979; Artru and Mekhfi 1990; Jaffe and Ji 1991]
[The third (leading twist) quark distribution function: transversity distribution]

Transverse spin naturally couples to transverse momenta. Many possible correlations between $k_T$, $S_T$ and $S_{qT}$ [Mulders et al., Kotzinian, ...: 1993-98]

$k_T$-dependent distributions (TMDs) give rise to single-spin asymmetries and azimuthal asymmetries in unpolarized reactions

Experiments in the last decade [HERMES, COMPASS, JLab] have shown that the asymmetries generated by the TMDs are sizable

Hadroproduction experiments [FNAL, RHIC] have provided beautiful data but their phenomenology is unclear
Transverse polarization $\equiv$ Transversity

A chirally-odd distribution:

$$h_1(x) = \int \frac{d\xi^-}{4\pi} e^{ixP^+\xi^-} \langle P, S|\bar{\psi}(0)\gamma^+\gamma_\perp\gamma_5\psi(\xi)|P, S\rangle \bigg|_{\xi^+=\xi_T=0}$$

[In light-cone gauge, $A^+ = 0$, Wilson line reduces to $1$]

To observe transversity, helicity must flipped twice, so one needs at least two hadrons (hadron-hadron scattering or semi-inclusive DIS).

No gluonic transversity distribution for spin-$\frac{1}{2}$ hadrons.

Non-singlet type evolution known up to NLO
Transverse-momentum distributions (TMDs)

\( k_T \)-dependent quark correlation matrix:

\[
\Phi(x, k_T) = \int \frac{d\xi^-}{2\pi} \int \frac{d^2 \xi_T}{(2\pi)^2} e^{ixP^+\xi^-} e^{-i\mathbf{k}_T \cdot \xi_T} \langle P, S|\bar{\psi}(0) \mathcal{W}[0, \xi] \psi(\xi)|P, S\rangle|_{\xi^+ = 0},
\]

SIDIS : \( \mathcal{W}[0, \xi] = \mathcal{W}^-[0, \infty] \mathcal{W}^T[0_T, \infty_T] \mathcal{W}^T[\infty_T, \xi_T] \mathcal{W}^-[\infty, \xi] \)
Transverse links survive in light-cone gauge and describe final-state interactions that generate “T-odd” TMDs [Brodsky, Hwang & Schmidt 2002; Belitsky, Ji & Yuan 2003].

**Leading-twist structure of the $k_T$-dependent quark correlator:**

$$
\Phi(x, k_T) = \frac{1}{2} \left\{ f_1 \gamma_+ - f_{1T}^{+} \frac{\varepsilon_{ij} k_{Ti} S_{Tj}}{M} \gamma_+ + \left( S_L g_{1L} + \frac{k_T \cdot S_T}{M} g_{1T} \right) \gamma_5 \gamma_+ \\
+ h_{1T} \frac{[S_T, \gamma_+] \gamma_5}{2} + \left( S_L h_{1L}^{+} + \frac{k_T \cdot S_T}{M} h_{1T}^{+} \right) \frac{[k_T, \gamma_+] \gamma_5}{2M} + i h_{1}^{+} \frac{[k_T, \gamma_+]}{2M} \right\}
$$

8 independent TMDs, three of which survive upon $k_T$ integration.
“T-odd” distributions $f_{1T}^\perp$ and $h_1^\perp$

$f_{1T}^\perp$ and $h_1^\perp$ measure T-odd correlations: $(\hat{P} \times k_T) \cdot S_T$ and $(\hat{P} \times k_T) \cdot S_{qT}$

Sivers distribution function

$$f_{q/p}^\uparrow(x, k_T) - f_{q/p}^\downarrow(x, -k_T) = 2 \frac{(k_T \times \hat{P}) \cdot S_T}{M} f_{1T}^\perp(x, k_T^2)$$

Azimuthal asymmetry of unpolarized quarks inside a transversely polarized proton

Boer-Mulders distribution function

$$f_{q/p}^\uparrow(x, k_T) - f_{q/p}^\downarrow(x, k_T) = \frac{(k_T \times \hat{P}) \cdot S_{qT}}{M} h_1^\perp(x, k_T^2)$$

Spin asymmetry of transversely polarized quarks inside an unpolarized proton

Due to the Wilson line structure, time reversal invariance does not imply $f_{1T}^\perp = 0$, but rather [Collins 2002]

$$f_{1T}^\perp(x, k_T^2)_{\text{SIDIS}} = -f_{1T}^\perp(x, k_T^2)_{\text{DY}}$$
Fragmentation functions

Unpolarized fragmentation function $D_1$ (couples to $f_{1T}^\perp$)

Collins fragmentation function

$$D_{h/q^\uparrow}(z, P_{hT}) - D_{h/q^\perp}(z, P_{hT}) = 2 \left( \frac{\kappa_T \times P_{hT}}{z M_h} \cdot S_{qT} \right) H_1^\perp(z, P_{hT}^2)$$

Fragmentation of transversely polarized quarks into an unpolarized hadron

Collins function $H_1^\perp$ couples either to transversity $h_1$ (in transversely polarized SIDIS), or to the Boer-Mulders function $h_1^\perp$ (in unpolarized SIDIS)
Focus on three distribution functions: $h_1$ (transversity), $h_1^\perp$ (Boer-Mulders), $f_{1T}^\perp$ (Sivers). The first two combine in SIDIS with $H_1^\perp$ (Collins).

Enforcing physical bounds (positivity, Soffer’s inequality) reduces the number of parameters.

Transverse-momentum dependence: Gaussian type (supported by lattice studies (see talk by Musch) and phenomenological analyses of SIDIS and DY [Schweitzer et al.]).

High-$x$ tails and antiquark distributions largely unconstrained.

Processes:

- $e p^{\uparrow} \rightarrow e' \pi X$ (Collins and Sivers effects with different angular distributions)
- $e p \rightarrow e' \pi X, p p \rightarrow \mu^+ \mu^- X$ (azimuthal asymmetries, Boer-Mulders effect)
- $e^+ e^- \rightarrow \pi \pi X$ (azimuthal asymmetries, Collins effect)
SIDIS cross section: 18 structure functions

\[
\frac{d^6\sigma}{dx_B\,dy\,dz_h\,d\phi_h\,dP_{h\perp}^2\,d\phi_S} = \frac{\alpha^2_{\text{em}}}{x_B\,y\,Q^2} \left\{ (1 - y + \frac{1}{2} y^2) F_{UU,T} + (1 - y) F_{UU,L} \right. \\
+ (2 - y) \sqrt{1 - y} \cos \phi_h F_{UU}^{\cos \phi_h} + (1 - y) \cos 2\phi_h F_{UU}^{\cos 2\phi_h} + \lambda_\ell y \sqrt{1 - y} \sin \phi_h F_{LU}^{\sin \phi_h} \\
+ S_\parallel \left[ (2 - y) \sqrt{1 - y} \sin \phi_h F_{UL}^{\sin \phi_h} + (1 - y) \sin 2\phi_h F_{UL}^{\sin 2\phi_h} \right] \\
+ S_\parallel \lambda_\ell \left[ y(1 - \frac{1}{2} y) F_{LL} + y \sqrt{1 - y} \cos \phi_h F_{LL}^{\cos \phi_h} \right] \\
+ S_\perp \left[ \sin(\phi_h - \phi_S) \left( (1 - y + \frac{1}{2} y^2) F_{UT,T}^{\sin(\phi_h - \phi_S)} + (1 - y) F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
+ (1 - y) \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + (1 - y) \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \\
+ (2 - y) \sqrt{1 - y} \sin \phi_S F_{UT}^{\sin \phi_S} + (2 - y) \sqrt{1 - y} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\
+ S_\perp \lambda_\ell \left[ y(1 - \frac{1}{2} y) \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + y \sqrt{1 - y} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
+ S_\perp \lambda_\ell \left[ y(1 - \frac{1}{2} y) \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \right\}
\]
SIDIS asymmetries:

\[
A^{w(\phi_h, \phi_S)}(x_B, y, z_h, P_{h\perp}) \equiv 2 \frac{\int d\phi_h \int d\phi_S w(\phi_h, \phi_S) d\sigma(\phi_h, \phi_S)}{\int d\phi_h \int d\phi_S d\sigma(\phi_h, \phi_S)} = K(y) \frac{F^{w(\phi_h, \phi_S)}}{F_{UU}}
\]

Structure functions in the extended parton model:

**Collins**

\[
F^{\sin(\phi_h + \phi_S)}_{UT} = C \left[ -\frac{\hat{h} \cdot \kappa_T}{M_{h}} h_1 H_1^\perp \right]
\]

**Sivers**

\[
F^{\sin(\phi_h - \phi_S)}_{UT,T} = C \left[ -\frac{\hat{h} \cdot k_T}{M} f_{1T}^\perp D_1 \right]
\]

**Boer – Mulders**

\[
F^{\cos 2\phi_h}_{UU} = C \left[ -\frac{2(\hat{h} \cdot k_T)(\hat{h} \cdot \kappa_T) - k_T \cdot \kappa_T}{MM_h} h_1^\perp H_1^\perp \right]
\]

**Cahn**

\[
F^{\cos 2\phi_h}_{UU, \text{Cahn}} = \frac{M^2}{Q^2} C \left[ \frac{(2(\hat{h} \cdot k_T)^2 - k_T^2)}{M^2} f_1 D_1 \right]
\]
Extended parton model is the zeroth-order approximation of the TMD factorization theorem, valid for \( P_{h \perp} (Q_T) \ll Q \) [Ji, Ma, Yuan]:

\[
F \sim \int d^2 k_T \int d^2 \kappa_T \int d^2 l_T \, \delta^2 (k_T - \kappa_T + l_T + q_T) \\
\times f(x_B, k_T^2) \hat{H}(Q^2) \, U(l_T^2) \, D(z_h, \kappa_T^2)
\]

At high transverse momenta, \( P_{h \perp} (Q_T) \gg \Lambda_{QCD} \), the twist-three collinear factorization theorem holds [Qiu, Sterman]:

\[
d\sigma \sim \int \frac{dx}{x} \int \frac{dz}{z} \, \delta \left( \frac{Q_T^2}{Q^2} - \left( 1 - \frac{x}{x_B} \right) \left( 1 - \frac{z}{z_h} \right) \right) \\
\times \left[ x \frac{d G_F(x, x)}{dx} \hat{H} + G_F(x, x') \hat{H}' \right] D(z) + \ldots
\]

In the intermediate region, \( \Lambda_{QCD} \ll P_{h \perp} (Q_T) \ll Q \), there is a relation between \( k_T \)-moments of TMDs and quark-gluon correlation functions

\[ \rightarrow \text{Evolution equations of TMDs (see Cherednikov’s talk)} \]
Collins asymmetries in SIDIS

Combined fit of SIDIS (HERMES, COMPASS) and $e^+e^-$ (Belle) [Anselmino et al.]

\[ A_{\text{Coll}}^{\pi^+} (p) \sim e_u^2 h_1^u H_1^{\text{fav}} + e_d^2 h_1^d H_1^{\text{unf}} \]

\[ A_{\text{Coll}}^{\pi^-} (p) \sim e_u^2 h_1^u H_1^{\text{unf}} + e_d^2 h_1^d H_1^{\text{fav}} \]

$A_{\text{Coll}}^{\pi^+} > 0$, $A_{\text{Coll}}^{\pi^-} < 0$: consistent with $h_1^u$ positive, $h_1^d$ negative

$|A_{\text{Coll}}^{\pi^-}| \sim |A_{\text{Coll}}^{\pi^+}|$ implies $|H_1^{\text{unf}}| \sim |H_1^{\text{fav}}|$
Extraction of \textit{tranversity} [Anselmino et al.]

\[ h_1^q(x) = N x^\alpha (1 - x)^\beta [f_1^q(x) + g_1^q(x)], \] Gaussian $k_T$ dependence, no antiquark

Red line: transversity; Dashed line: helicity; Blue line: Soffer bound ($Q^2 = 2.4$ GeV$^2$)
Prediction for COMPASS data with transversely polarized proton [Anselmino et al.]
Sivers asymmetries in SIDIS

[Anselmino et al.]

Burkardt sum rule \( \sum_{a=q,\bar{q},g} \int_0^1 dx \, f_{1T}^{(1)}(x) a(x) = 0 \) saturated by quarks and antiquarks: no room for gluons

Signs and magnitudes \( (f_{1T}^u \sim -f_{1T}^d) \) in agreement with chiral models, impact-parameter approach and large \( N_c \)
Boer-Mulders asymmetries in SIDIS [VB, Melis, Prokudin]

Three sources of $\langle \cos 2\phi \rangle$: pQCD (negligible), Boer-Mulders and Cahn (both relevant)

Prediction of $A_{\pi^-}^{\cos 2\phi} > A_{\pi^+}^{\cos 2\phi}$ confirmed by data (first evidence of BM)

Signs and magnitudes ($h_{1u}^{1} \sim 2f_{1T}^{u}$, $h_{1d}^{1} \sim -f_{1T}^{d}$) in agreement with theoretical expectations (impact-parameter + lattice, large $N_c$)
At small $Q_T$, the $\cos 2\phi$ asymmetry in DY production ($\nu$ parameter) is dominated by the Boer-Mulders contribution

$$\frac{d^6\sigma_{UU}}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{6sQ^2} \left\{ (1 + \cos^2 \theta) W_{UU}^1 + \sin^2 \theta W_{UU}^2 + \sin 2\theta \cos \phi W_{UU}^{\cos \phi} + \sin^2 \theta \cos 2\phi W_{UU}^{\cos 2\phi} \right\}$$

$$\nu = \frac{2 W_{UU}^{\cos 2\phi}}{W_{UU}^1 + W_{UU}^2} \quad W_{UU}^{\cos 2\phi} = \frac{1}{3} C \left[ \frac{2(\hat{h} \cdot k_{1T})(\hat{h} \cdot k_{2T}) - k_{1T} \cdot k_{2T}}{M_1 M_2} \right] h_1^+ \tilde{h}_1^+$$

E866/NuSea data vs. [VB, Melis & Prokudin] (see also [Lu & Schmidt])
Implement evolution of transverse momentum distributions

Move towards global fits (take all perturbative and non-perturbative effects into account, fit simultaneously polarized and unpolarized cross sections, etc.)

Enlarge datasets with:
  - More SIDIS data (JLab, EIC): neutron target, wider $x$ range, etc.
  - Polarized DY measurements
    Probe various combinations of $h_1$, $f_{1T}^\perp$ and $h_1^\perp$
    No fragmentation functions involved
# Future Drell-Yan experiments

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<tr>
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Direct determination of $h_1$ from $p^\uparrow p^\uparrow$, $p^\uparrow \bar{p}^\uparrow$

More information on $h_1^\perp$ and $f_{1T}^\perp$ from $pp^\uparrow$

Determination of antiquark distributions

Test of modified universality: SIDIS = - DY

$A_{TT}$ at NLO [VB et al.]

Sivers asymmetry [Anselmino et al.]
Summary

- Transverse-spin and transverse-momentum physics is a rapidly evolving field.
- Many important experimental (single-spin asymmetries) and theoretical (identification of TMDs, relation with twist-three, evolution equations, lattice) results.
- Phenomenology is growing. It will come to maturity (global fits) with the advent of polarized DY data.
EXTRA SLIDES
Evolution of $h_1$ known at NLO. At small $x$ suppressed compared to helicity.
Semi-inclusive DIS

\[ l, l', S_\perp, \phi_S, P_{h,\perp}, P_h, \phi_h \]
\[ \pi^+ + S\phi + h\phi \sin (\pi A) \]

\[ \pi^- + S\phi + h\phi \sin (\pi A) \]

\[ x, z, P_T (GeV) \]

\[ \sin (\phi_n + \phi_S + \pi) \]
SSAs in inclusive hadroproduction

\[ p + p^\uparrow \rightarrow h + X \]