Forward collisions and spin effects in evaluating amplitudes

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Summary

- New energy window available at the Large Hadron Collider
- Total cross sections and phases form part of an early study
- Hadronic spin dependence in the pp differential cross section
- Vacuum polarization contributions to the photon propagator
- Luminosities from the elastic scattering of protons and ions
- Outlook and conclusions

Introduction

- Proton proton collisions probe analyticity and spin effects
- A dispersion relation violation may suggest a new mass scale
- Check Bourrely-Soffer-Wu and Kaidalov-Ter Martyrosian models
- Hadronic spin effects appear in $t = 0$ fermion fermion scattering
- Seek the spin implications when evaluating transition amplitudes
- Study forward spin dependence for cross section normalisation

Small angle collisions of protons and ions

The level of hadronic spin dependence is important in the context of

- Generalised parton distributions and nucleon form factors: Selyugin and Teryaev, arXiv:0712.1947 [hep-ph]
- Measuring the proton beam polarization at BNL RHIC, Makdisi, AIP Conf Proc 980
- Double Pomeron and multi Pomeron exchange processes, Troshin & Tyurin, Mod Phys Lett A23
- Spin dependent couplings of the Pomeron, Trueman, Phys Rev D77

Total cross sections and phases

Forward collision amplitudes form part of the early studies when a new energy window becomes available as is now provided by the Large Hadron Collider. At the interference invariant momentum transfer

$$t_c \;=\; -8\,\pi\,\alpha/\sigma_{\rm tot} \qquad \alpha \;=\; 1/137$$

the spin averaged electromagnetic and hadronic amplitudes are of comparable magnitude, Kopeliovich & Lapidus YaF 19. The value of $-t_c$ is expected to decrease at the LHC 7 – 14 TeV energies where

$$\sigma_{\rm tot} \;\approx\; 90 - 130 \text{ mb}$$

Higher order photon contributions are encoded in a spin independent

Coulomb phase multiplying electromagnetic amplitudes of exponent

$$\delta \;=\; -\alpha \,\ln\left|\,B\,t/2s \,+\, 4\,t/\Lambda^2\,\right| \,-\, \alpha\,\gamma$$

where B relates to the exponential t-behaviour of the hadronic differential cross section, Euler's constant is $\gamma = 0.5772\ldots$, and $\Lambda^2 = 0.71\,\mathsf{GeV}^2$ reflects the small momentum transfer dependence of

$$G_E(t) \;\approx\; G_M(t)/\mu \;\approx\; (1 - t/\Lambda^2)^{-2}$$

the Sachs electromagnetic proton form factors with $\mu = 2.7928$ as the magnetic moment of the proton. Observe that in pp collisions, by contrast with spin scalar scattering, the high energy element

$$(\alpha\,/\,t\,)\,(s - 2m^2)\,{F_1}^2(t)$$

of the spin averaged electromagnetic amplitude involves the Dirac form factor, NB, Gotsman, and Leader, Phys Rev D18 694

$$F_1(t) \ = \ \frac{G_E - G_M\, t/4m^2}{1 - t/4m^2}$$

The Dirac form factor has the following expansion in small $-t$, using the above approximation to the Sachs form factor involving Λ^2

$$F_1(t) \ \approx \ 1 + \left(\frac{2}{\Lambda^2} - \frac{\mu - 1}{4m^2} \right) t$$

so that an extra term appears in the interference element of the usual expression describing the differential cross section for elastic proton proton collisions in the interference region, BKLST Phys Rev D59.

The expansion including singular terms in t and constant terms is

$$\frac{16\pi}{\sigma_{\rm tot}^2}\frac{d\sigma}{dt}\,e^{-bt} \;=\; \frac{t_c^2}{t^2} - 2\,(\,\rho+\delta+\epsilon\,)\,\frac{t_c}{t} + 1+\rho^2$$

$$- \;\; 2\,t\left[\left(\tfrac{1}{2}\,\kappa\,t_c/t - {\rm Re}\,r_5\right)^2 + \left(\,{\rm Im}\,r_5\right)^2\right]\Big/ m^2$$

$$+ \;\; \left[2\,\Delta\sigma_{\rm T}^{\,2}\left(1+\rho_2^2\right) \;+\; \Delta\sigma_{\rm L}^{\,2}\left(1+\rho_-^{\,2}\right)\right]/\,4\,\sigma_{\rm tot}^{\,2}$$

where δ is the Bethe phase and $\kappa = 1.7928$ is the anomalous moment of the proton. The extra interference term includes the hadronic slope

$$\epsilon \;=\; \left(\frac{B}{2} - \frac{4}{\Lambda^2} + \frac{\mu-1}{2m^2}\right) t_c \;=\; \frac{B-9.23}{2}\,t_c$$

The last term is a sum of forward transverse and longitudinal hadronic double helicity flip contributions involving respective real part to imaginary part parameters ρ_2 & ρ_- and total cross section differences

$$\Delta\sigma_{\rm T} \;=\; \sigma_{\uparrow\downarrow} \,-\, \sigma_{\uparrow\uparrow} \qquad\qquad \Delta\sigma_{\rm L} \;=\; \sigma_{\rightleftarrows} \,-\, \sigma_{\rightrightarrows}$$

It has been found non-zero by Bourrely Soffer Wray, Nucl Phys B77. As a fraction of the forward imaginary spin averaged amplitude, the kinematically scaled single helicity flip hadronic amplitude r_5 obeys

$$|\, r_5 \,| \;<\; 0.19 \qquad\qquad (6.8 \text{ GeV})$$

resulting from a polarised jet study of the single spin asymmetry $A_{\rm N}$ at RHIC, Alekseev et al, Phys Rev D79. Expressions for electromagnetic helicity amplitudes in spin-$\frac{1}{2}$ spin-$\frac{1}{2}$ elastic collisions were also used by the same group in establishing a bound at the higher energy

$$|\, r_5 \,| \; < \; 0.05 \qquad (13.7 \text{ GeV})$$

This and other studies at Fermilab, Akchurin et al, Phys Rev D51, and at Brookhaven National Lab, Bültmann et al, Phys Lett B632, have indicated that the absolute value of the single helicity flip hadronic amplitude at another two energies is limited by

$$|\, r_5 \,| \; < \; 0.41 \;\; (19.4 \text{ GeV}); \;\; 0.86 \;\; (200 \text{ GeV}).$$

A preliminary result of Sverida, Guryn (STAR) from this meeting is

$$|\, r_5 \,| \; < \; 0.06 \;\; (200 \text{ GeV}).$$

Neglecting the apparently small single helicity flip hadronic amplitude the differential cross section near the interference region now reads

$$\frac{16\pi}{\sigma_{\rm tot}^{\,2}}\,\frac{d\sigma}{dt}\,e^{-bt} \;=\; \frac{t_c^{\,2}}{t^{\,2}} \,-\, 2\,(\,\rho+\delta+\varepsilon\,)\,\frac{t_c}{t} \,+\, 1+\rho^2$$

$$+\;\left[2\,\Delta\sigma_{\rm T}^{\,2}\left(1+\rho_2^2\right) \,+\, \Delta\sigma_{\rm L}^{\,2}\left(1+\rho_-^{\,2}\right)\right]/\,4\,\sigma_{\rm tot}^{\,2}$$

where, incorporating the single helicity flip electromagnetic amplitude,

$$\varepsilon \;=\; \left(\frac{B}{2}-\frac{4}{\Lambda^2}+\frac{\mu^2-1}{4\,m^2}\right)t_c \;=\; \frac{B-7.40}{2}\,t_c$$

The extra term $(\mu^2-1)/4\,m^2\,t_c = 1.93\,t_c$ would contribute to ρ.

Transverse spin total cross section differences have also been found

$$\Delta\sigma_{\rm T} = \begin{cases} -1.26 \pm 0.88 \text{ mb } (\ \ 6.8 \text{ GeV}) \\ \ \ 0.02 \pm 0.23 \text{ mb } \ (13.7 \text{ GeV}) \\ -0.04 \pm 0.50 \text{ mb } \ (200 \ \text{ GeV}) \end{cases}$$

and also the absolute value of transverse contributions are available from recent measurements of $A_{{\rm N}N}$ at RHIC Alekseev et al, PR D79

$$\frac{\Delta\sigma_{\rm T}}{\sigma_{\rm tot}}\left(1 + \rho_2^2\right)^{1/2} < \begin{cases} 0.103 \ \ (\ \ 6.8 \text{ GeV}) \\ 0.017 \ \ (13.7 \text{ GeV}) \end{cases}$$

Enhancement of the forward elastic differential cross section above that expected from estimates of dispersion relation and optical theorem values may result from the presence of hadronic spin dependence. Bounds on such enhancements are now emerging. In addition to the above transverse effects, contributions from a longitudinal total cross section difference measured at Fermilab

$$\Delta\sigma_{\mathrm{L}} = 0.040 \pm 0.048 \pm 0.052 \text{ mb} \ \ (200 \text{ GeV})$$

are negligible at 200 GeV but nothing is currently known about the corresponding real part or the values at higher energies in the longitudinal case. The Relativistic Heavy Ion Collider at BNL is an almost unique position to continue its spin programme of polarized proton and ion scattering to understand the contributions of spin dependence to forward collisions and diffraction at higher energies.

Vacuum polarization effects

An electron loop in the photon propagator induces a vacuum polarization contribution West & Yennie, Phys Rev 172, that increases the coupling α to $\alpha(1+\Delta v)$ with a t-dependence given by Källén (QED)

$$\Delta v = \frac{\alpha}{3\,\pi}\left[\frac{1}{3} + \left(3 - \frac{1}{\tau}\right)\left(\frac{1}{3}\,\tau + \frac{1}{5}\,\tau^2 + \frac{1}{7}\,\tau^3 + \cdots\right)\right]$$

in the case $|\tau| < 1$ where $\tau^{-1} = 1 - 4m_e^2/t$. The correction Δv has a t-dependence (Jauch & Rohrlich) given for $-t \gg m_e^2$ by

$$\Delta v^{(e)} = \frac{\alpha}{3\pi}\left(\ln\left|\frac{t}{m_e^2}\right| - \frac{5}{3} - \frac{6m_e^2}{t} + \cdots\right)$$

For $0 < -t \ll m_\mu^2$ negligible contributions to the vacuum polarization result from muon and higher mass pairs. At $\sqrt{s} = 546$ GeV, for example, the vacuum polarization contribution to the real part parameter ρ(pp) would be (West & Yennie)

$$\Delta v = 0.005$$

in the interference region of proton proton elastic scattering and have the opposite sign for antiproton proton collisions. Further enhancement to ρ(pp), in addition to those arising from a consideration of electromagnetic form factor and helicity fip amplitudes, therefore results from a study of vacuum polarization contributions to the photon propagator.

Conclusions

- Elastic scattering studies of protons and ions at small angles are important in the evaluation of beam luminosities
- Polarization measurements indicate hadronic spin dependence
- Analyticity of amplitudes probes a possible high mass scale
- Vacuum polarisation contributions should not be neglected
- Low momentum transfer studies have implications for diffraction