Fluctuations, Saturation and Diffractive Excitation

Christoffer Flensburg

Department of Theoretical Physics
Lund University

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Work done with Gösta Gustafson.
Introduction
The Lund Dipole Cascade Model
Fluctuations

Content

➤ Diffractive excitation: Good–Walker vs Tripple-Regge.

➤ Extending Good–Walker into the Tripple-Regge regime.

➤ Fluctuations, Saturation and Diffractive excitation in DIPSY.

➤ Comparison of Good–Walker and Tripple-Regge.
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Diffractive excitation

Often treated by two mechanisms:

- Low mass: Good–Walker
  - Incoming projectile superposition of interaction eigenstates.
  - Diffractive excitation from fluctuations in interaction eigenvalues.
  - Used mainly at low excited masses $M_X$.

- Medium and high mass: Tripple-Regge
  - Reggeons that couple to the target, projectile and each other.
  - Diffractive excitation from the Tripple-Regge couplings.
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Extending Good–Walker to high $M_X$

The problem:
- Fluctuations in the pomeron ladders are not included.
- BFKL ladders have very large fluctuations.

The solution, Lund Dipole Cascade model:
- Use BFKL dipoles in transverse space.
- Generates cascades for the entire rapidity range.
- Gets the BFKL fluctuations naturally.
- Saturation limits fluctuations in dense cascades.
- Monte Carlo implementation: DIPSY.
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Good–Walker formalism

Incoming mass eigenstate:

\[ \psi_{\text{in}} = \sum_n c_n \phi_n \]

\( \phi_n \) diffractive scattering eigenstates, with eigenvalues \( T_n \).

Elastic amplitude:

\[ A_{\text{el}} = \langle \psi_{\text{in}} | T | \psi_{\text{in}} \rangle = \sum_n c_n^2 T_n = \langle T \rangle \]

\[ d\sigma_{\text{el}} / d^2 b = \langle T \rangle^2 \]

The diffractive amplitude to the state \( X \) can also be calculated.

\( \sigma_{\text{diff ex}} \) comes from the fluctuations in \( T \)!

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The diffractive eigenstates


- A colour singlet exchange will give a rapidity gap.

virtual cascade  inelastic int.  elastic scatt.  diffractive exc.
The Lund Dipole Cascade

The Lund model is a generalisation of Mueller’s dipole model, with the following improvements:

- Includes NLL BFKL effects.
- Includes non-linear saturation effects in the cascade.
- Includes confinement effects.

Needs valence states to start the cascade:

- Protons are model with a triangle of dipoles. Soft QCD!
- Virtual photons in DIS is a single dipole. Hard QCD!

Implemented in a Monte Carlo: DIPSY.
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Example cascade

Evolves both valence states up to a certain rapidity $Y_0$, where $T$ is calculated.
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What does Diffraction mean for us?

Needed frame: many different definitions around. This is for our model.

- Depends on interaction frame $Y_0$!
- Colour singlet exchange in interaction frame, ie rapidity gap covering $Y_0$.
- Includes all excited masses $M_X < e^{Y_0} \cdot \text{GeV}$.
- No overlapping excitations in double diffraction.

Now ready to study diffraction in the Lund Dipole Model!
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Diffractive cross sections

- Cross sections are integrated over all lower masses $M_X < e^{Y_p} \cdot \text{GeV}$.
- Differential cross section $d\sigma/dY$ is the derivative.
- Gray areas are data from Tevatron.
Fluctuations

The origin of the diffractive excitation in the Good–Walker formalism lies in the distribution of interaction probabilities.

- Each pair of cascade produce a single interaction amplitude $F$.
- Multiple interaction amplitude $T = 1 - e^{-F}$.
- The fluctuations $\langle T^2 \rangle - \langle T \rangle^2$ determines the diffractive cross section.
- Average is over cascades.
- Study the frequency $P$ of different $F$ and $T$ to understand diffraction.
- Look in mid-rapidity, ie $Y_0$ in com rest frame.
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P(F) in DIS ($Q^2 = 14$)

- $F \ll 1$, so $F \approx T$.
- $P(F)$ can be parametrised by a power $F^{-\rho}$, freeze-out for low $F$.
- $\rho$ between 1.6 and 1.8 for different $b$, $W$ and $Q$.
- Wide distribution $\rightarrow$ high $\sigma_{\text{diff ex}} / \sigma_{\text{tot}}$. 
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- \( F \) not smaller than \(1 \rightarrow \) saturation important.
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The Lund Dipole Cascade Model

Fluctuations

Comparison with Tripple-Regge

DIS

pp

Impact Parameter Profile

P(T) in pp

High F pushed in just below $T = 1$.

Low $b$, high energy peaked at $T = 1$: small fluctuations.

High $b$ peaked at $T = 0$: small fluctuations.

Medium $b$ evenly spread out from 0 to 1: large fluctuations.
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Fluctuations
Comparison with Triple-Regge

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Impact parameter profile in $pp$

- Saturation suppresses fluctuations at low $b$:
  $\langle T^2 \rangle \approx \langle T \rangle^2 \approx 1$.
- Diffractive excitation lives in a ring where $T \approx 0.5$.
- Slower asymptotic energy growth $\sim \ln s$ compared to total and elastic $\sim \ln^2 s$. 
Compare to Goulianos

DIPSY isn’t supposed to be used below $\sim 100$ GeV, but...

- At $\sim 20$ GeV, the unitary effect is weak, even at $b = 0$. 

![Graph showing total single diffraction cross section vs. $\sqrt{s}$ (GeV)]
Tripple-Regge Formalism

- **Bare pomerons:**
  - Pomerons couple to the target and projectile with strength $\beta(t)$.
  - Tripple-pomeron coupling $g_{3P}(t)$.
  - Pomeron trajectory $\alpha(t) = 1 + \varepsilon + \alpha' t$

- \[ \sigma_{\text{tot}} = \beta^2(0) s^{\alpha(0)-1} \equiv \sigma_0^{p\bar{p}} s^\varepsilon \]

- \[ \frac{d\sigma_{\text{el}}}{dt} = \frac{1}{16\pi} \beta^4(t) s^{2(\alpha(t)-1)} \]

- \[ M_X^2 \frac{d\sigma_{\text{SD}}}{dtd(M_X^2)} = \frac{1}{16\pi} \beta^2(t) \beta(0) g_{3P}(t) \left( \frac{s}{M_X^2} \right)^{2(\alpha(t)-1)} (M_X^2)^\varepsilon \]

- Higher order effects and saturation different in different models.
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- Higher order effects and saturation different in different models.
Strategy for comparison

► *Compare only to bare pomerons to avoid model dependence.*

► Use DIPSY without saturation: no saturation in cascade, and use $F$ rather than $T$ as amplitude.

► Look at energy dependence of total, elastic and diffractive cross sections.

► See if the Tripple-Regge equations can describe DIPSYs energy dependence.

► Tune $\varepsilon, \alpha', \beta(t)$ and $g_{3P}(t)$ to fit to DIPSY.

► Compare parameters to other Tripple-Regge models.
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Results

\[ \alpha(0) = 1 + \varepsilon = 1.21, \quad \alpha' = 0.2 \text{ GeV}^{-2}, \]
\[ \sigma_{0}^{p\bar{p}} = \beta^{2}(0) = 12.6 \text{ mb}, \quad b_{0,\text{el}} = 8 \text{ GeV}^{-2}, \]
\[ g_{3P}(t) = \text{const.} = 0.3 \text{ GeV}^{-1}. \]
Comparison to other Models bare parameters

  - $\alpha(0) = 1.21$, $\alpha' = 0.2\text{GeV}^{-1}$

  - $\alpha(0) = 1.3$, $\alpha' \leq 0.05\text{GeV}^{-1}$

  - $\alpha_h(0) = 1.35$, $\alpha'_h = 0.08\text{GeV}^{-1}$
  - $\alpha_s(0) = 1.15$, $\alpha'_s = 0.14\text{GeV}^{-1}$

  - $\alpha(0) = 1.335$, $\alpha' = 0.01\text{GeV}^{-1}$

  - $\alpha(0) = 1.12$, $\alpha' = 0.22\text{GeV}^{-1}$

  - $\alpha(0) = 1.11$, $\alpha' = 0.26\text{GeV}^{-1}$
Summary

- **Good–Walker** can normally only describe low mass excitations.
- The **Lund Dipole Cascade** model can extend the Good–Walker formalism to large masses for $\gamma^* p$ and $pp$.
- Fluctuations, and thus $\sigma_{\text{diff ex}}$, are strongly suppressed by unitarity in $pp$.
- Diffractive excitation in $pp$ is an expanding ring in $b$.
- **DIPSY** without saturation agrees very well with Tripple-Regge for a bare pomeron.
- corresponds to simple pole with $\alpha(0) = 1.21$ and $\alpha' = 0.2$. (With saturation cross section grows $\sim s^{0.1}$ up to 50 TeV.)
- Produces exclusive final states as well (soon to be published).
CDF

Pseudorapidity distribution and $N_{ch}$ in towards region.

Pseudorapidity distribution at $\sqrt{s} = 1800$ GeV

$dN_{ch}/d\eta$

CDF data
MC (TestFull1800)

$N_{ch}$ (toward) for min-bias

$N_{ch}$

MC/data

CDF data
MC (TestFull1800)
CDF

Angular distribution and multiplicity frequency.

\[ \langle p_{\text{sum}}^{\perp} \rangle \text{ vs. } \Delta \phi \text{ from leading jet } (p_{\text{lead}}^{\perp} > 2 \text{ GeV}) \]

\[ \text{Charged multiplicity at } \sqrt{s} = 1800 \text{ GeV, } |\eta| < 1, p_{T} > 0.4 \text{ GeV} \]
Track $p_T$ and $\sum E_T$ distributions.

**CDF data**

**MC (TestFull1800)**

$\frac{d^3\sigma}{dp_T d\eta d\phi} / (mb/GeV^2)$

$p_T, |\eta| < 1, p_\perp > 0.4 \text{ GeV}$

$\frac{d^3\sigma}{dE_T d\eta d\phi} / (mb/GeV)$

$\sum E_T, |\eta| < 1$
Rapidity distribution and Multiplicity frequency.
Heavy Ions

Central Au-Au collision in $y-x_T$ space.
Some sample results

\( \text{pp and } \gamma^*p: \text{ total, elastic and diffractive cross section.} \)
Collide a single dipole with a cascade of nucleons. Can again use same tools as for $pp$.

Here follows a sample event just as proof of concept:
Or better seen in a $y$-$p_T$ plot:
Evolution in rapidity

A colour dipole emits a gluon in transverse space with probability

\[
\frac{dP}{dy} = \frac{\bar{\alpha}}{2\pi} d^2 r_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2}
\]

Equivalent to LL BFKL.
Interaction

A Born level calculation gives the collision amplitude for a pair of dipoles from different states:

\[ f_{ij} = \frac{\alpha^2 s}{2} \ln^2 \left( \frac{r_{13}r_{24}}{r_{14}r_{23}} \right). \]

With the eikonal approximation, the total unitarised probability then becomes

\[ t \equiv 1 - e^{-\sum f_{ij}}. \]
Modifications in DIPSY

Energy conservation

- Keep track of $p_\mu$ for all partons.
- Small dipoles $\leftrightarrow$ high $p_T$.
- Gives dynamic cutoff for small emissions.

Non-linear $2 \rightarrow 2$ swing:

- Saturation in evolution.

Confinement

- Supression of too large dipoles.