Fluctuations, Saturation and Diffractive Excitation

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Work done with Gösta Gustafson.

Content

- Diffractive excitation: Good–Walker vs Tripple-Regge.
- Extending Good–Walker into the Tripple-Regge regime.
- Fluctuations, Saturation and Diffractive excitation in DIPSY.
- Comparison of Good–Walker and Tripple-Regge.

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Diffractive excitation

Often treated by two mechanisms:

- Low mass: Good–Walker
  - Incoming projectile superposition of interaction eigenstates.
  - Diffactive excitation from fluctuations in interaction eigenvalues.
  - Used mainly at low excited masses M_X.
- Medium and high mass: Tripple-Regge
  - Reggeons that couple to the target, projectile and each other.
  - Diffractive excitation from the Tripple-Regge couplings.
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Extending Good–Walker to high M_X

- The problem:
  - Fluctuations in the pomeron ladders are not included.
  - BFKL ladders have very large fluctuations.

- The solution, Lund Dipole Cascade model:
  - Use BFKL dipoles in transverse space.
  - Generates cascades for the entire rapidity range.
  - Gets the BFKL fluctuations naturally.
  - Saturation limits fluctuations in dense cascades.
  - Monte Carlo implementation: DIPSY.

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Good–Walker formalism

Incoming mass eigenstate:

- $\Psi_{\text{in}} = \sum_n c_n \phi_n$

ϕ_n diffractive scattering eigenstates, with eigenvalues T_n.
Elastic amplitude:

- $A_{\text{el}} = \langle \Psi_{\text{in}} | T | \Psi_{\text{in}} \rangle = \sum_n c_n^2 T_n = \langle T \rangle$
- $d\sigma_{\text{el}}/d^2b = \langle T \rangle^2$

The diffractive amplitude to the state X can also be calcualated.
$\sigma_{\text{diff ex}}$ comes from the fluctuations in T!

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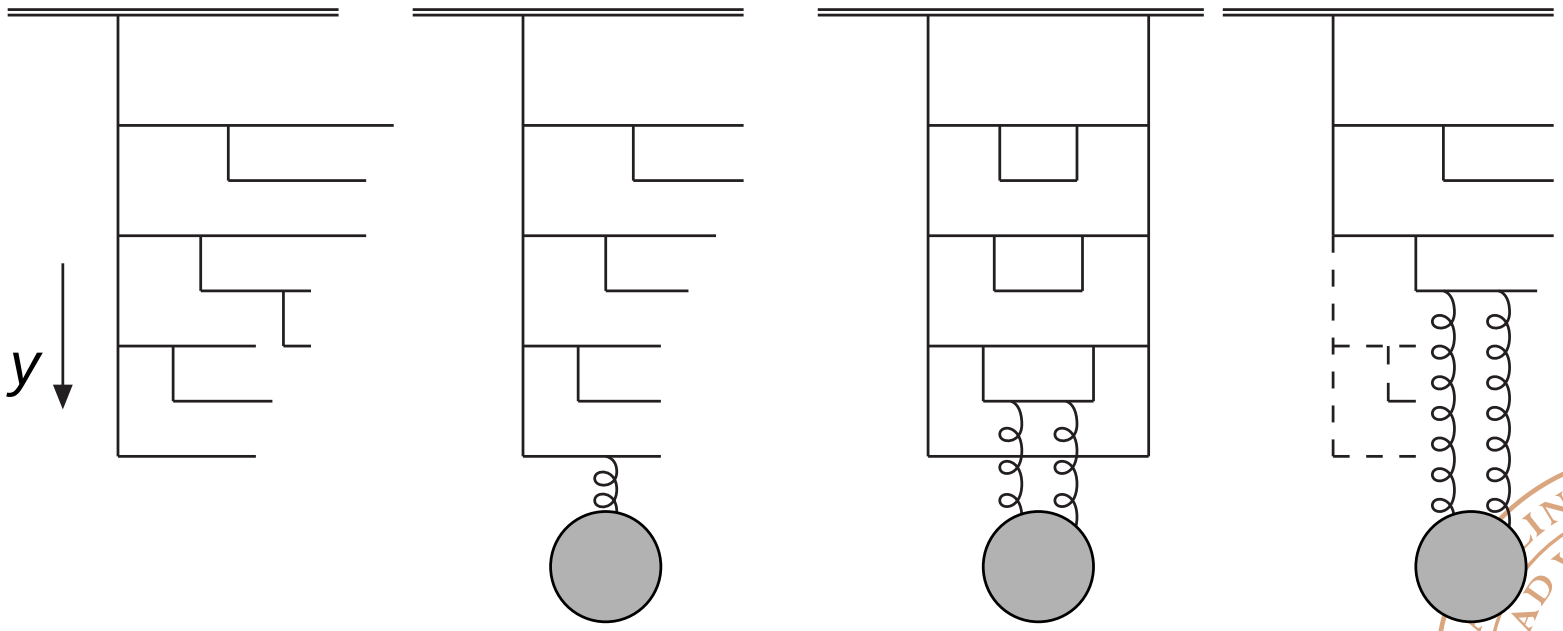
The diffractive eigenstates

Parton cascades: Miettinen-Pumplin (1978), Hatta *et al.* (2006). Use DIPSY to generate cascades.

- A colour singlet exchange will give a rapidity gap.



virtual cascade *inelastic int.* *elastic scatt.* *diffractive exc.*

The Lund Dipole Cascade

The Lund model is a generalisation of Mueller's dipole model, with the following improvements:

- Includes NLL BFKL effects.
- Includes non-linear saturation effects in the cascade.
- Includes confinement effects.

Needs valence states to start the cascade:

- Protons are model with a triangle of dipoles. Soft QCD!
- Virtual photons in DIS is a single dipole. Hard QCD!

Implemented in a Monte Carlo: DIPSY.

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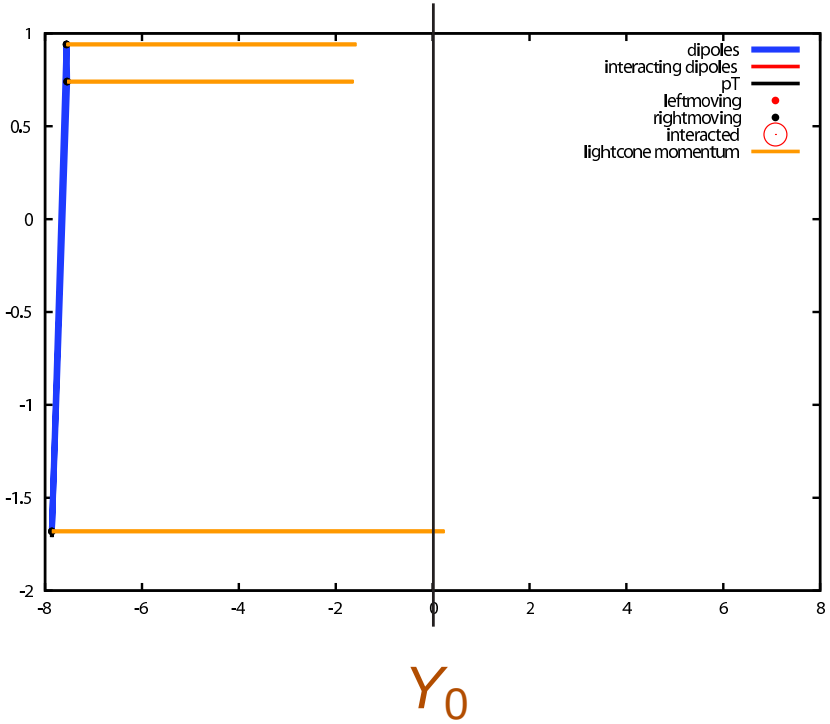
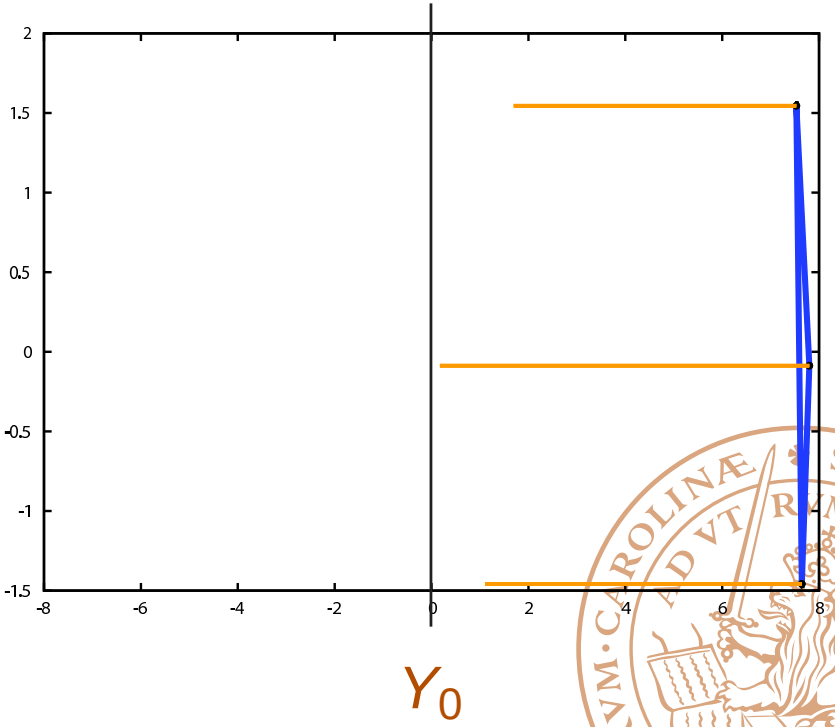
Example cascade

Evolves both valence states up to a certain rapidity Y_0, where T is calculated.





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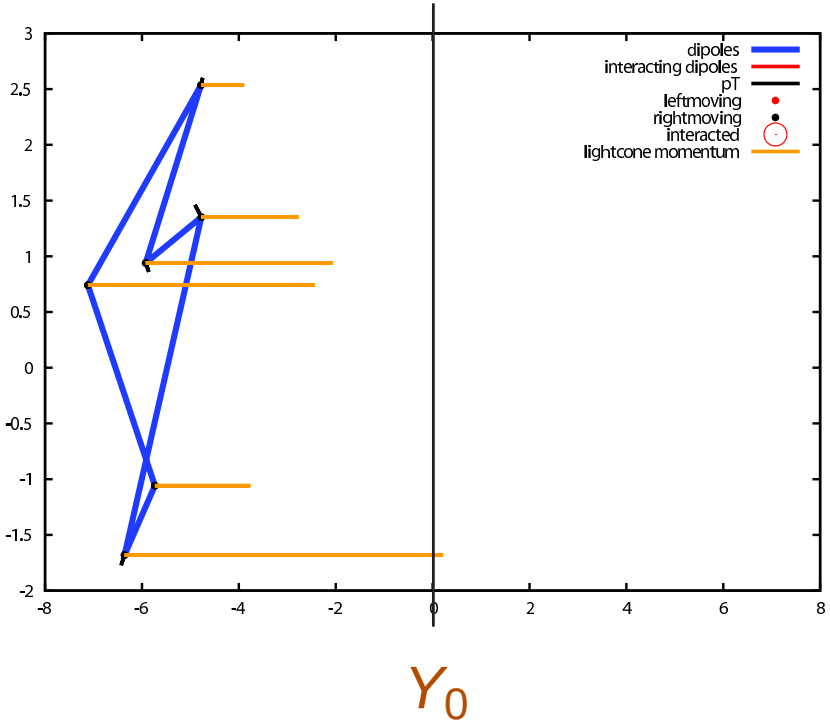

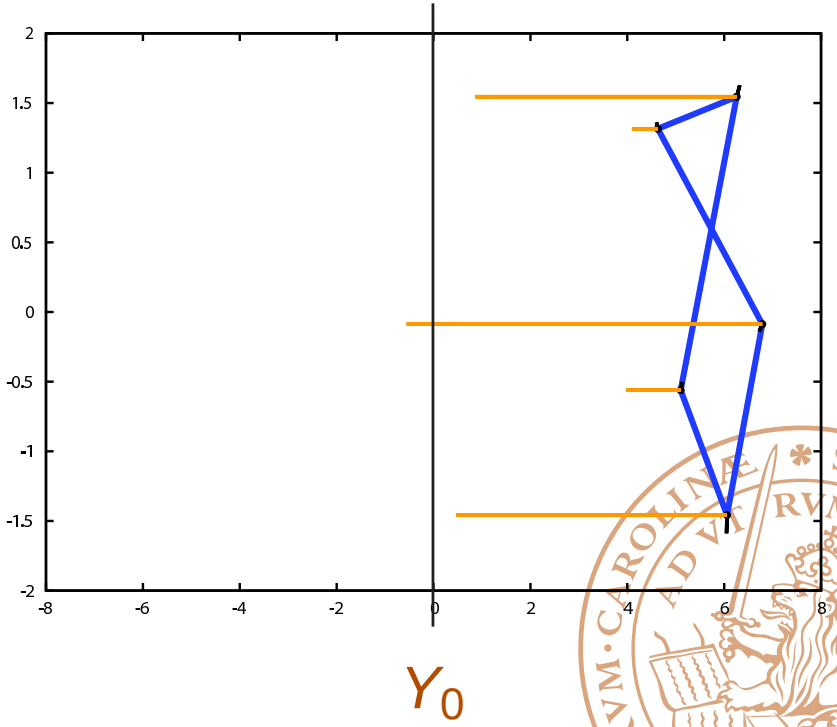

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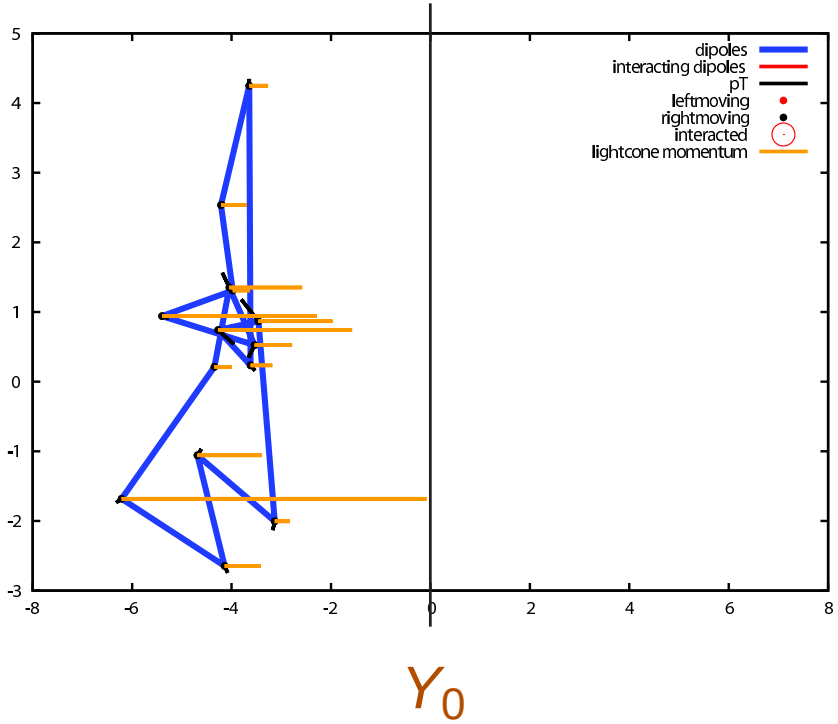

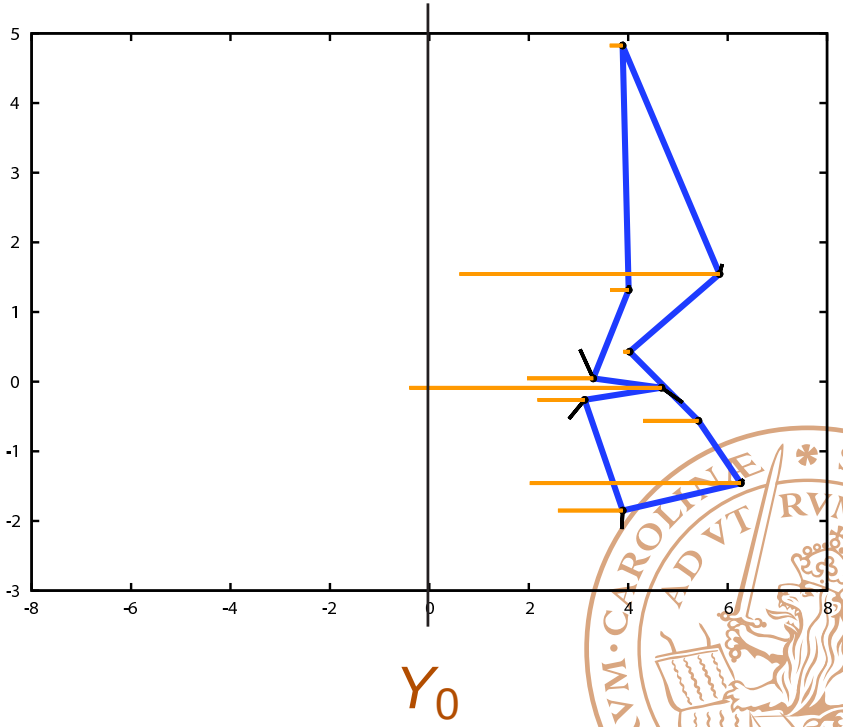

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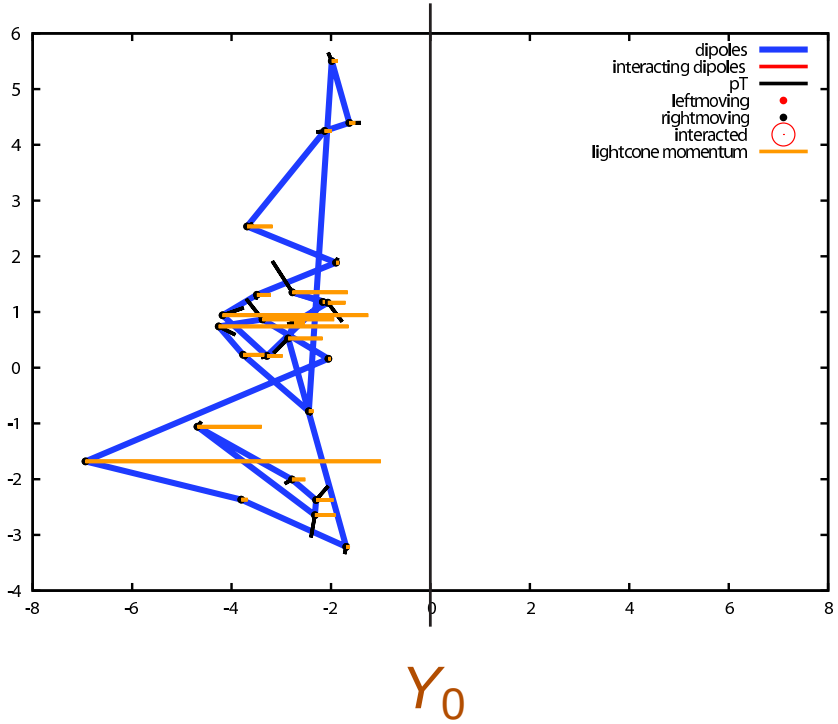

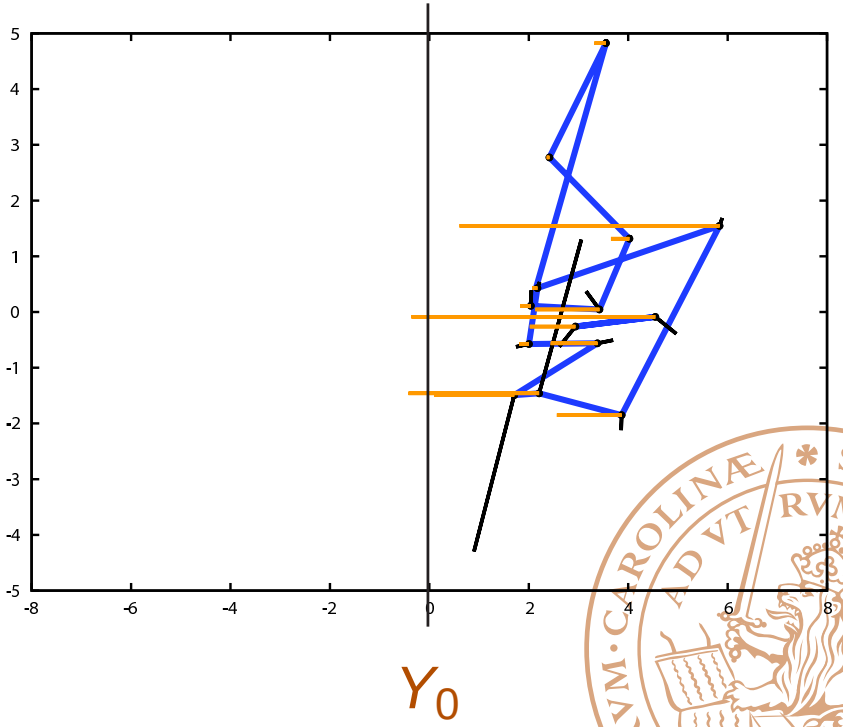

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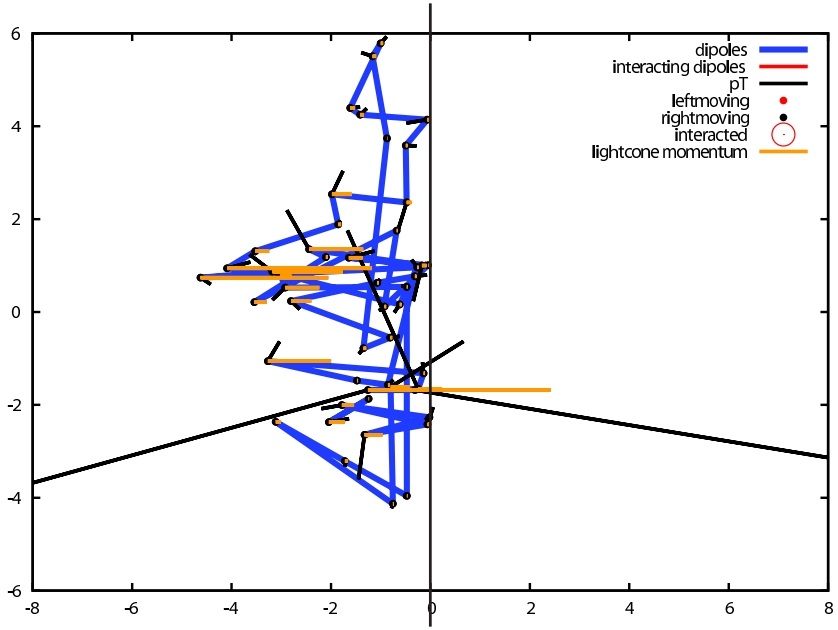

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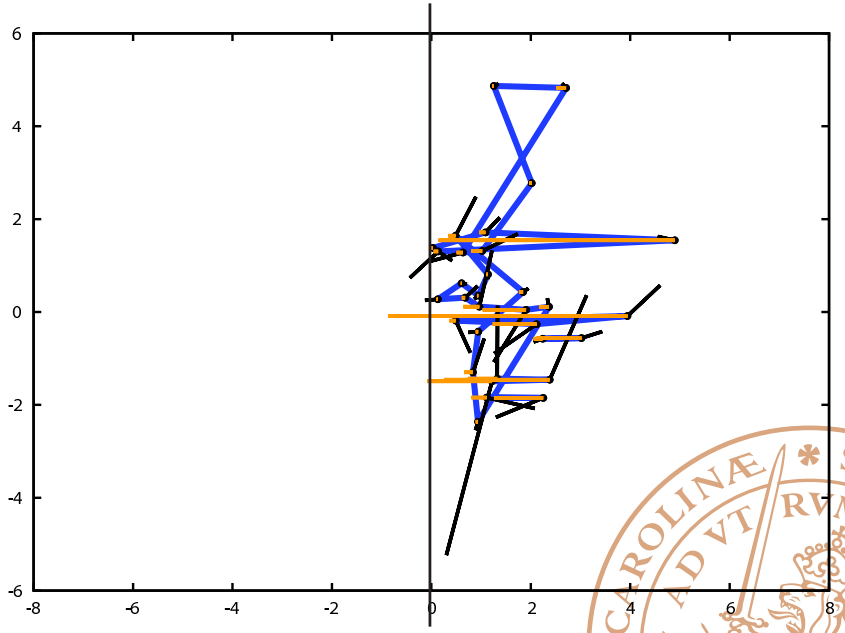

Y_0

What does Diffraction mean for us?

Needed frame: many different definitions around. This is for our model.

- Depends on interaction frame Y_0!
- *Colour singlet exchange in interaction frame, ie rapidity gap covering Y_0.*
- Includes all excited masses $M_X < e^{Y_0} \cdot GeV$.
- No overlapping excitations in double diffraction.

Now ready to study diffraction in the Lund Dipole Model!

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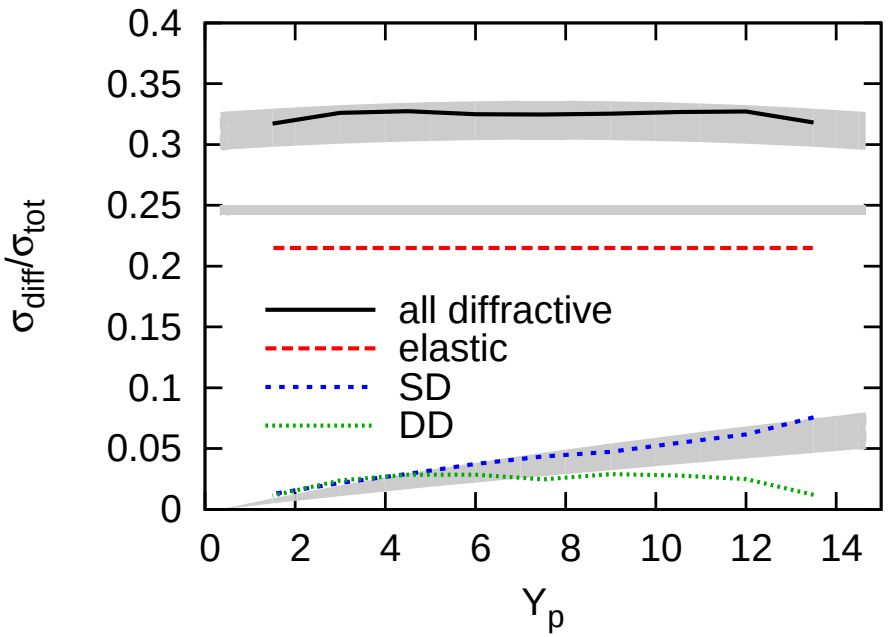
Diffractive cross sections



- Cross sections are integrated over all lower masses $M_X < e^{Y_p} \cdot GeV$.
- Differential cross section $d\sigma/dY$ is the derivative.
- Gray areas are data from Tevatron.

Fluctuations

The origin of the diffractive excitation in the Good–Walker formalism lies in the distribution of interaction probabilities.

- Each pair of cascade produce a single interaction amplitude F.
- Multiple interaction amplitude $T = 1 - e^{-F}$.
- The fluctuations $\langle T^2 \rangle - \langle T \rangle^2$ determines the diffractive cross section.
- Average is over cascades.
- Study the frequency P of different F and T to understand diffraction.
- Look in mid-rapidity, ie Y_0 in com rest frame.

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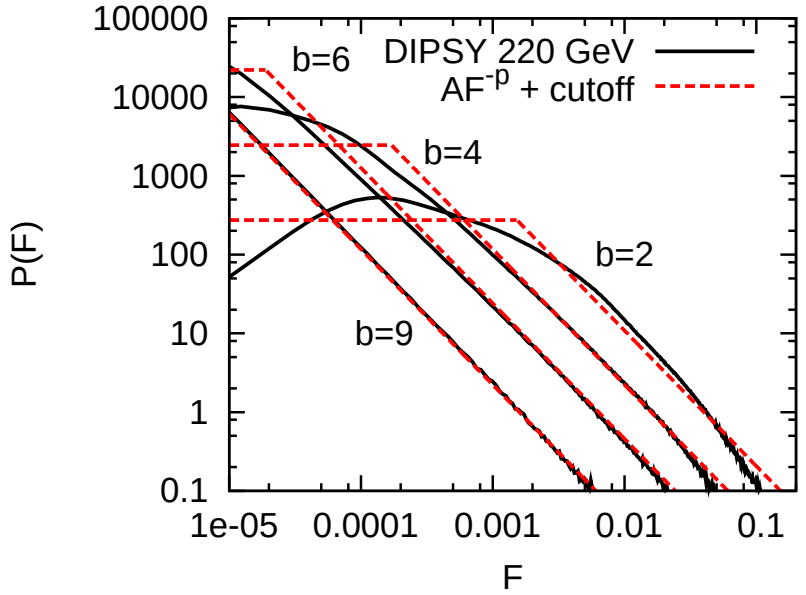
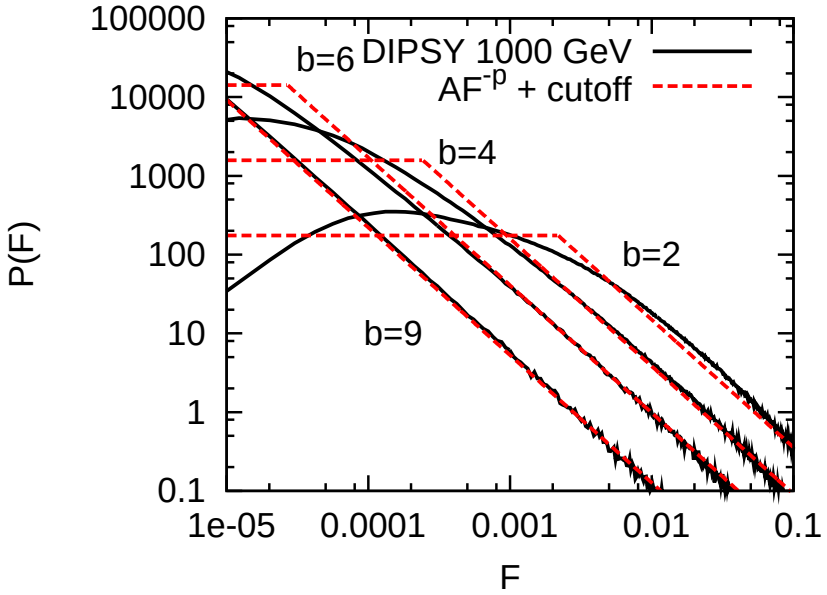
P(F) in DIS ($Q^2 = 14$)





- $F << 1$, so $F \approx T$.
- $P(F)$ can be parametrised by a power F^{-p}, freeze-out for low F.
- p between 1.6 and 1.8 for different b, W and Q.
- Wide distribution $\rightarrow$ high $\sigma_{\text{diff ex}}/\sigma_{\text{tot}}$.

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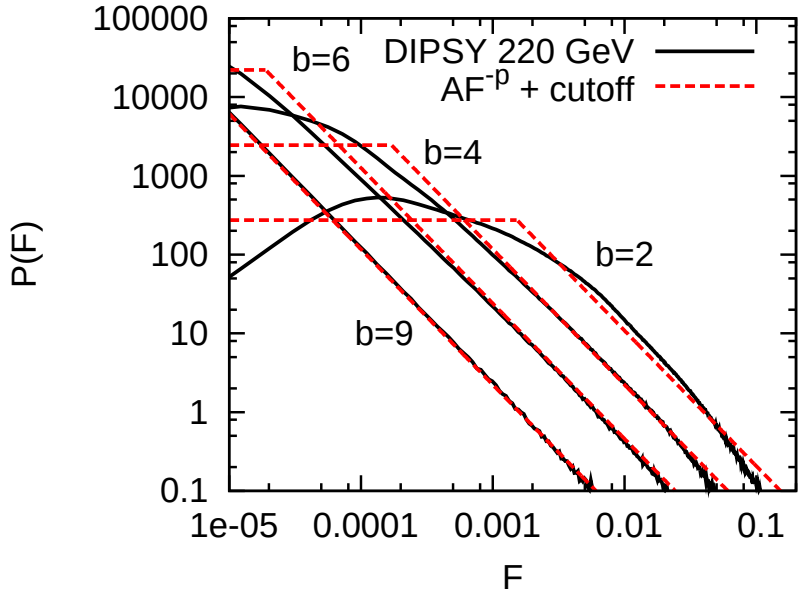

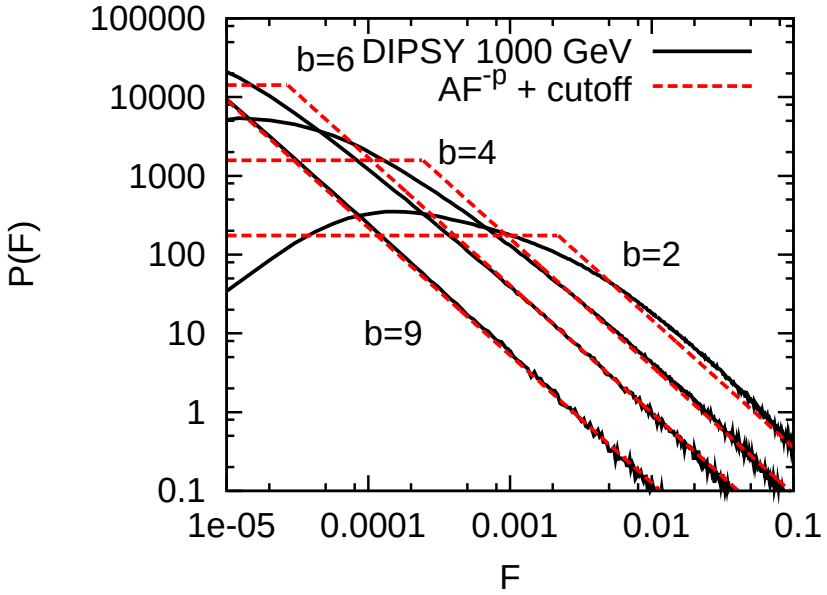

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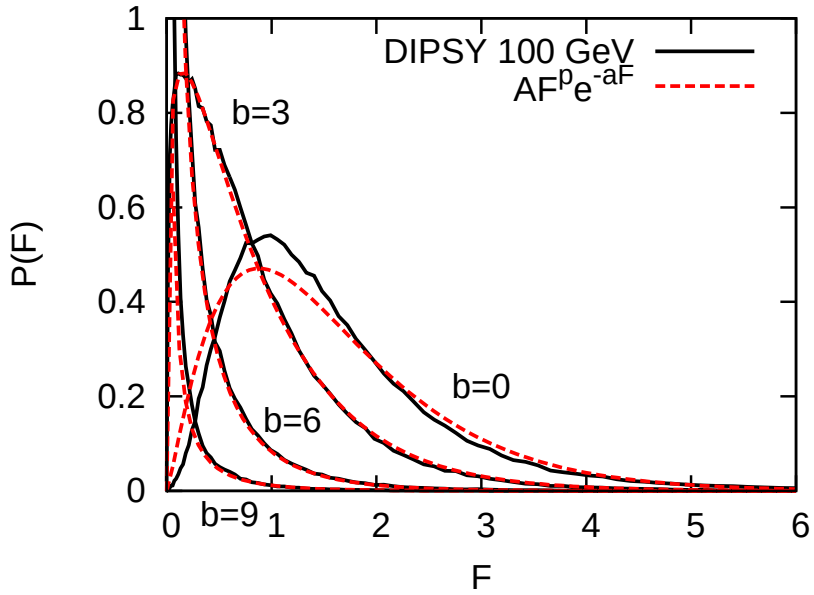

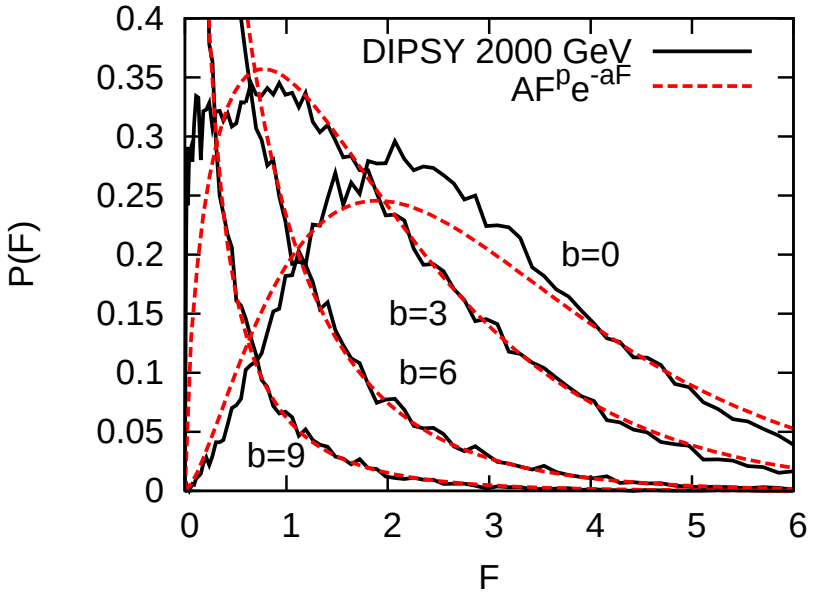

- $P(F)$ can be parametrised by $F^p e^{-aF}$.
- $a \sim 1$, p between -0.7 and 1.5.
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- F not smaller than 1 $\rightarrow$ saturation important.
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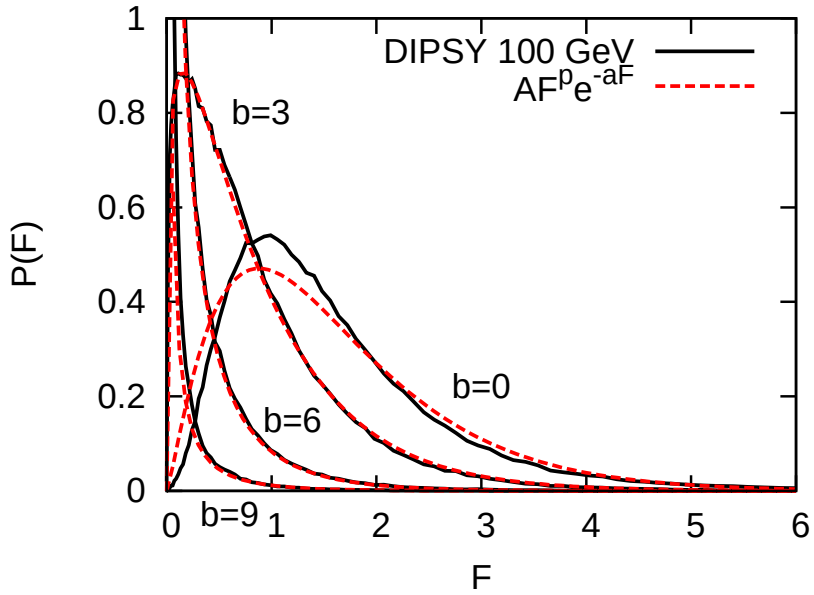

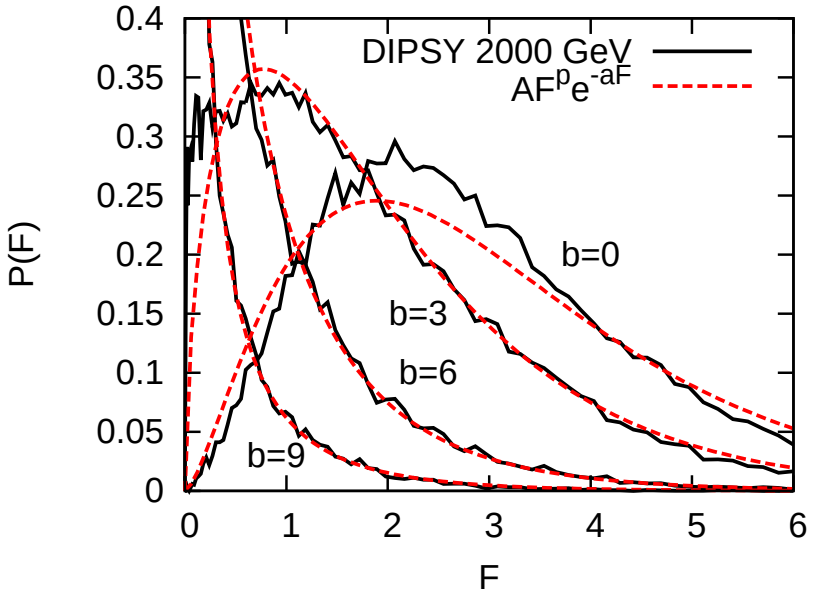

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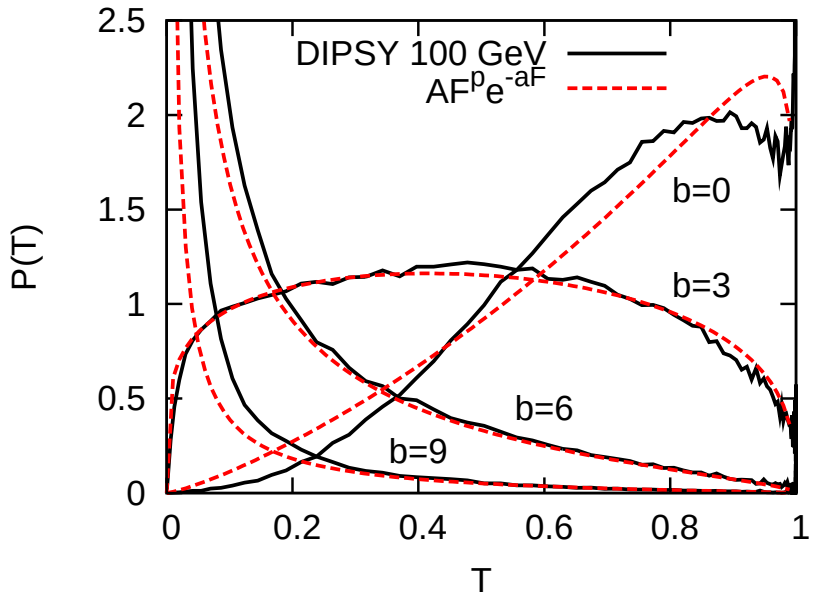

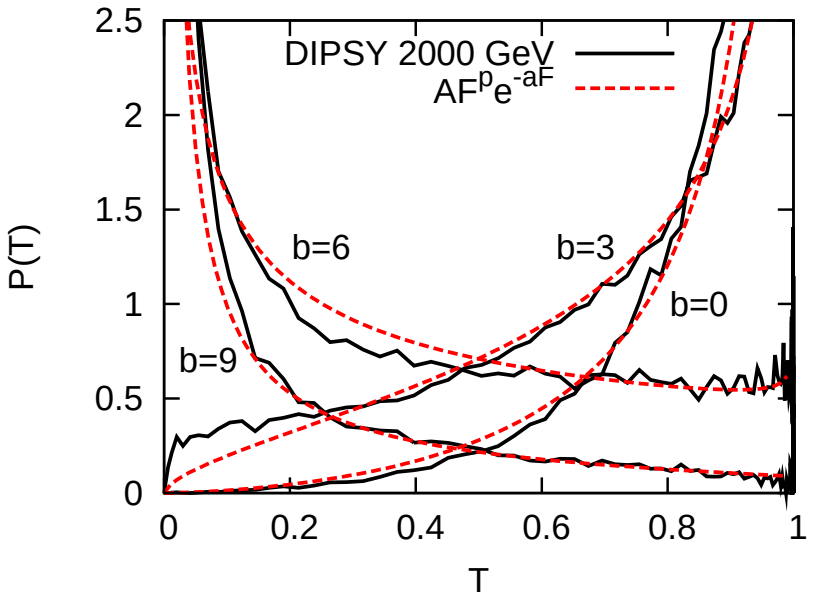

- High F pushed in just below $T = 1$.
- Low b, high energy peaked at $T = 1$: small fluctuations.
- High b peaked at $T = 0$: small fluctuations.
- Medium b evenly spread out from 0 to 1: large fluctuations.

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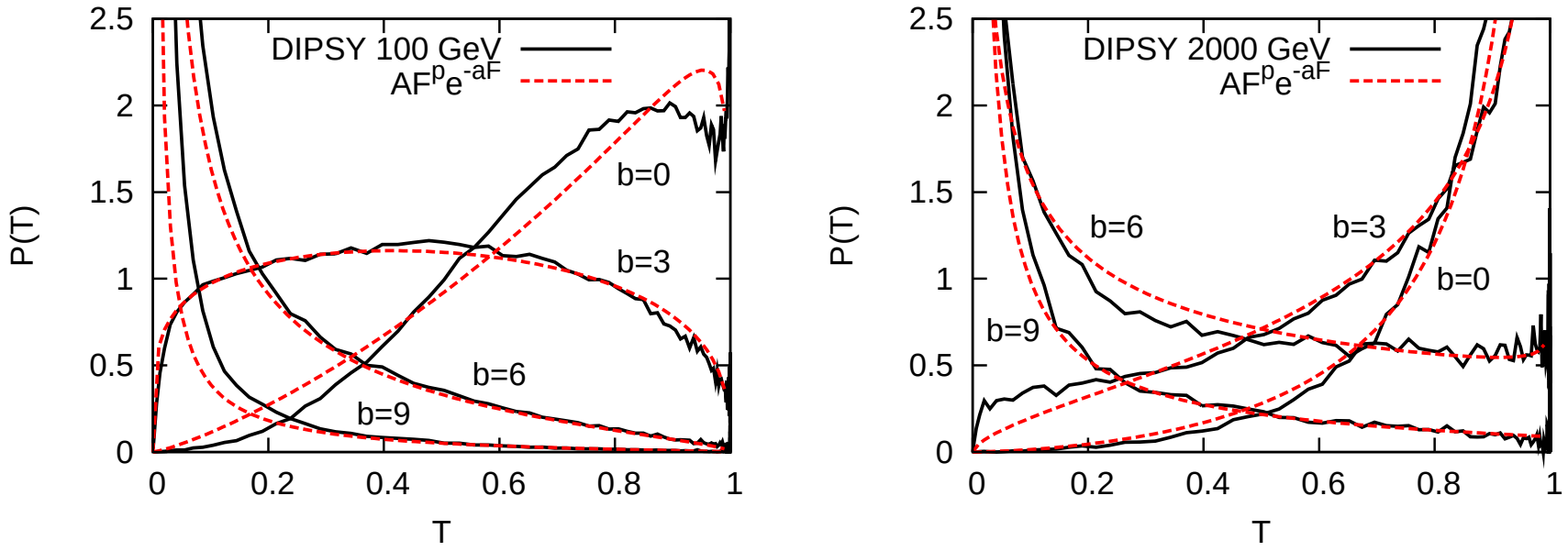

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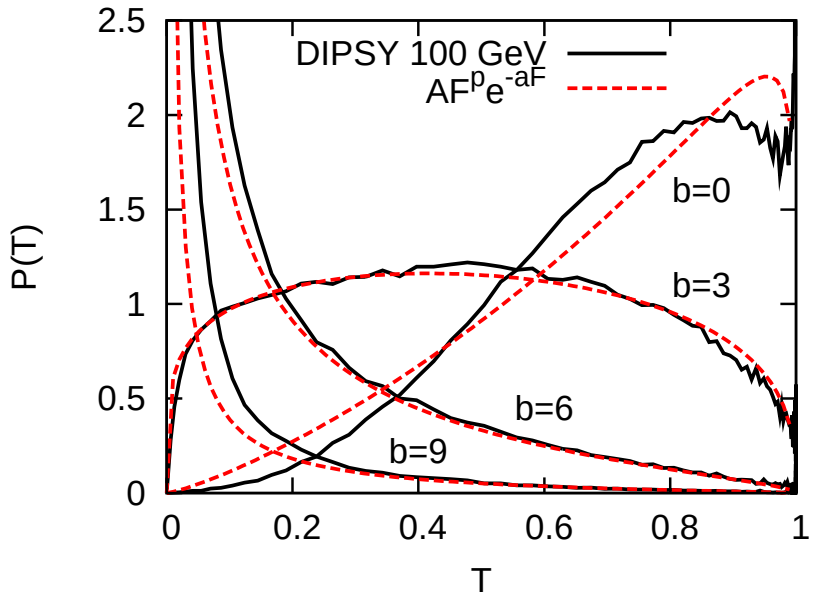

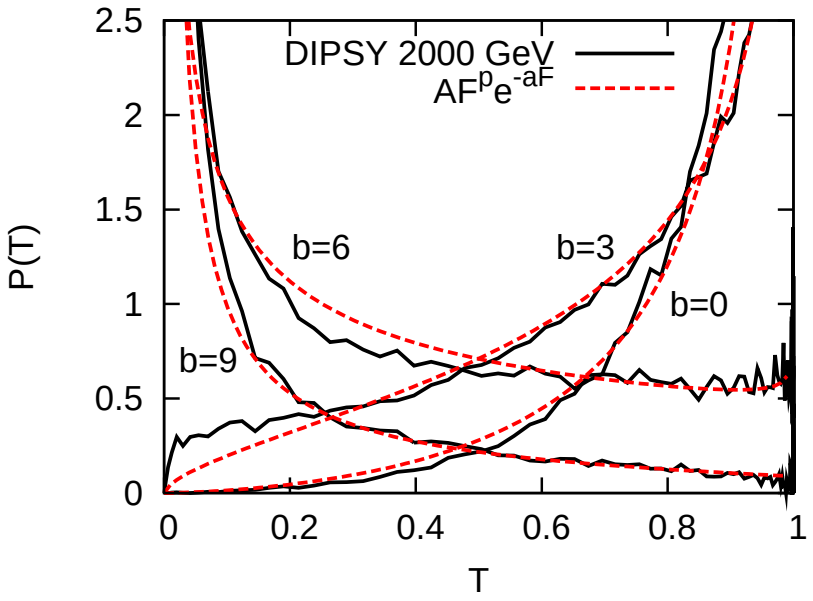

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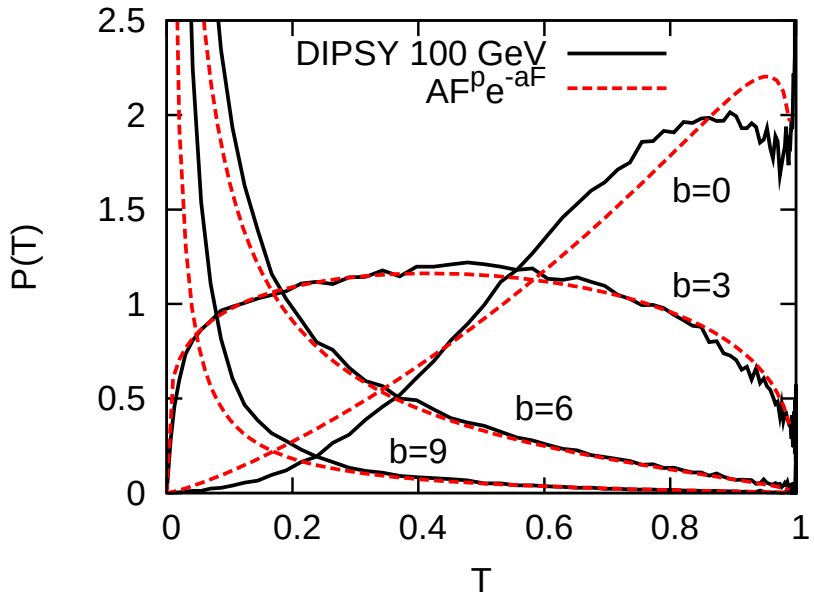

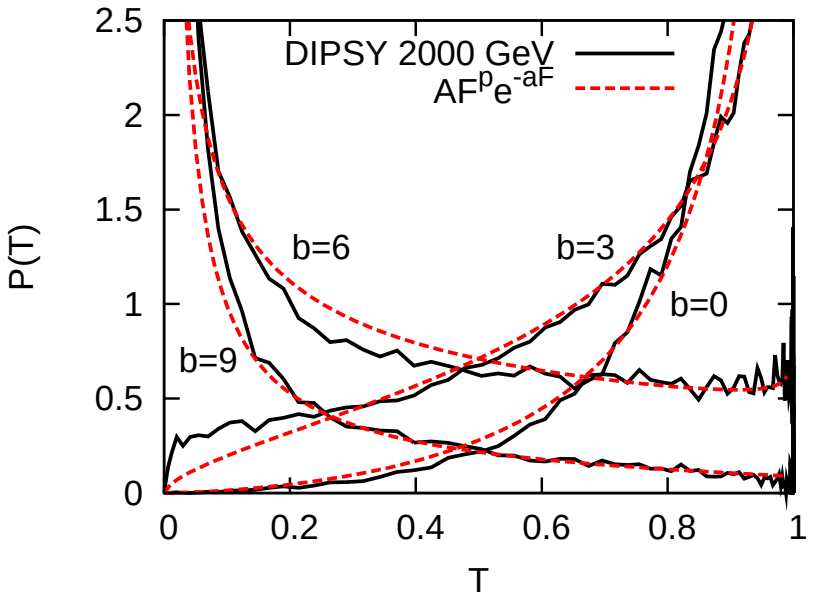

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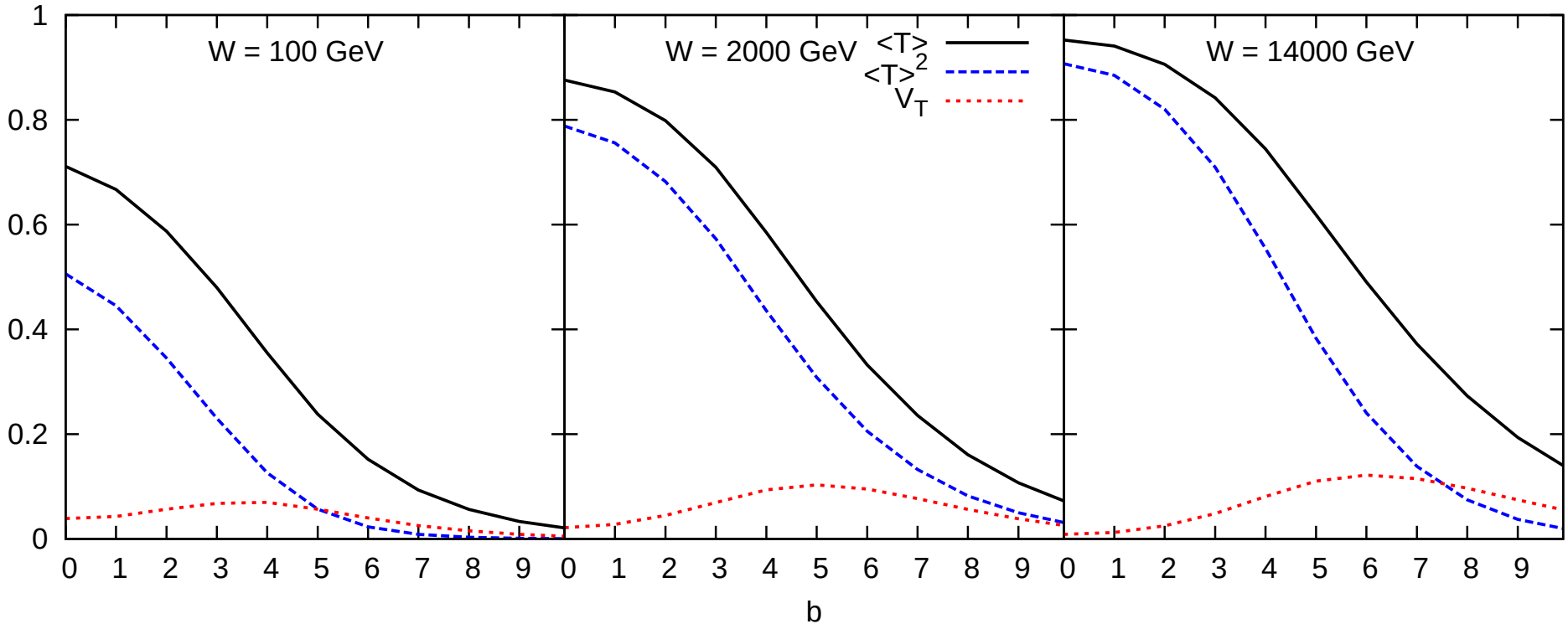
Impact parameter profile in *pp*

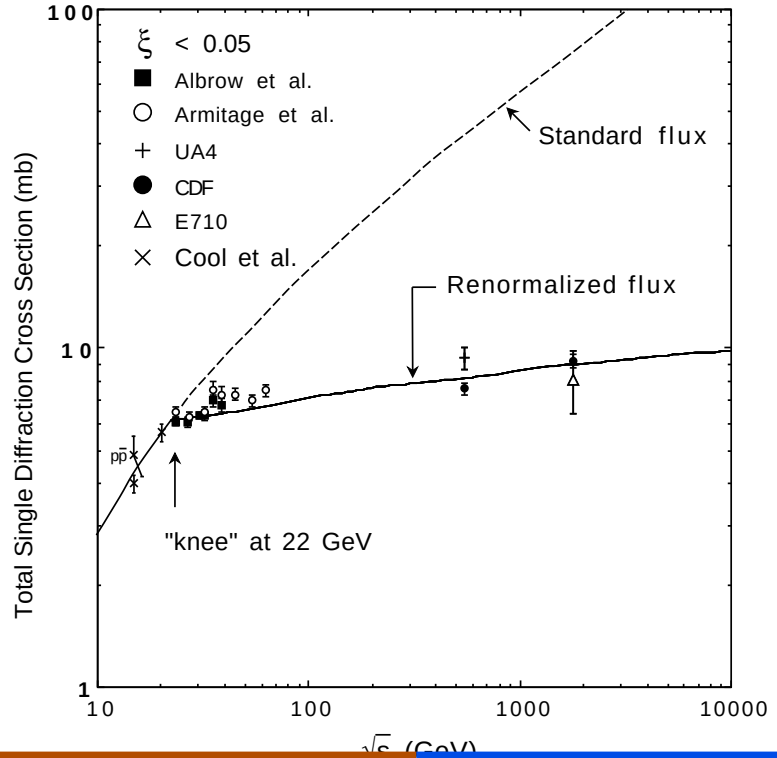


- Saturation suppresses fluctuations at low b: $\langle T^2 \rangle \approx \langle T \rangle^2 \approx 1$.
- Diffractive excitation lives in a ring where $T \approx 0.5$.
- Slower asymptotic energy growth $\sim \ln s$ compared to total and elastic $\sim \ln^2 s$.

Compare to Goulianos

DIPSY isn't supposed to be used below $\sim$ 100 GeV, but...

- At $\sim$ 20 GeV, the unitary effect is weak, even at $b = 0$.



Tripple-Regge Formalism

- Bare pomerons:
  - Pomerons couple to the target and projectile with strenght $\beta(t)$.
  - Tripple-pomeron coupling $g_{3P}(t)$.
  - Pomeron trajectory $\alpha(t) = 1 + \varepsilon + \alpha' t$
  - $\sigma_{\text{tot}} = \beta^2(0) s^{\alpha(0)-1} \equiv \sigma_0^{p\bar{p}} s^{\varepsilon}$
  - $\frac{d\sigma_{\text{el}}}{dt} = \frac{1}{16\pi}\beta^4(t) s^{2(\alpha(t)-1)}$
  - $M_X^2 \frac{d\sigma_{\text{SD}}}{dt d(M_X^2)} = \frac{1}{16\pi}\beta^2(t)\beta(0) g_{3P}(t) \left(\frac{s}{M_X^2}\right)^{2(\alpha(t)-1)} \left(M_X^2\right)^{\varepsilon}$
- Higher order effects and saturation different in different models.

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Strategy for comparison

- *Compare only to bare pomerons to avoid model dependence*.
- Use DIPSY without saturation: no saturation in cascade, and use F rather than T as amplitude.
- Look at energy dependence of total, elastic and diffractive cross sections.
- See if it the Tripple-Regge equations can describe DIPSYs energy dependence.
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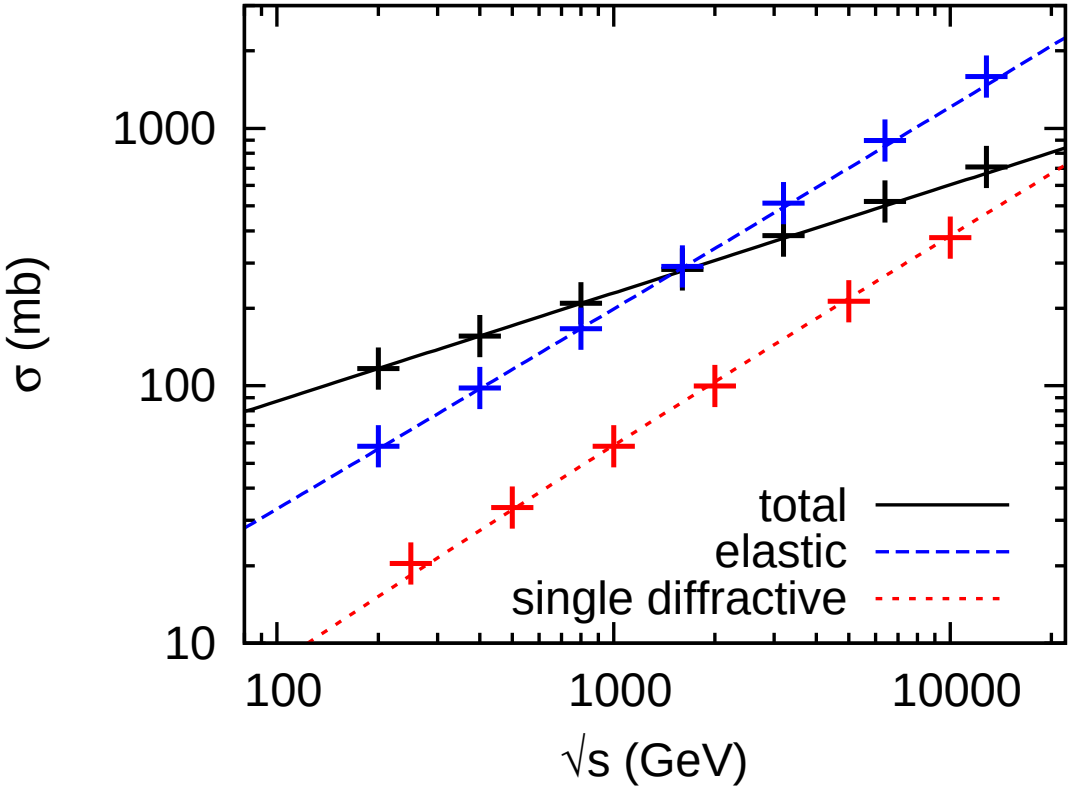
Results



$$\alpha(0) = 1 + \varepsilon = 1.21, \quad \alpha' = 0.2\,\mathrm{GeV}^{-2},$$
$$\sigma_0^{p\bar{p}} = \beta^2(0) = 12.6\,\mathrm{mb}, \quad b_{0,\mathrm{el}} = 8\,\mathrm{GeV}^{-2},$$
$$g_{3P}(t) = \text{const.} = 0.3\,\mathrm{GeV}^{-1}.$$

Comparison to other Models bare parameters

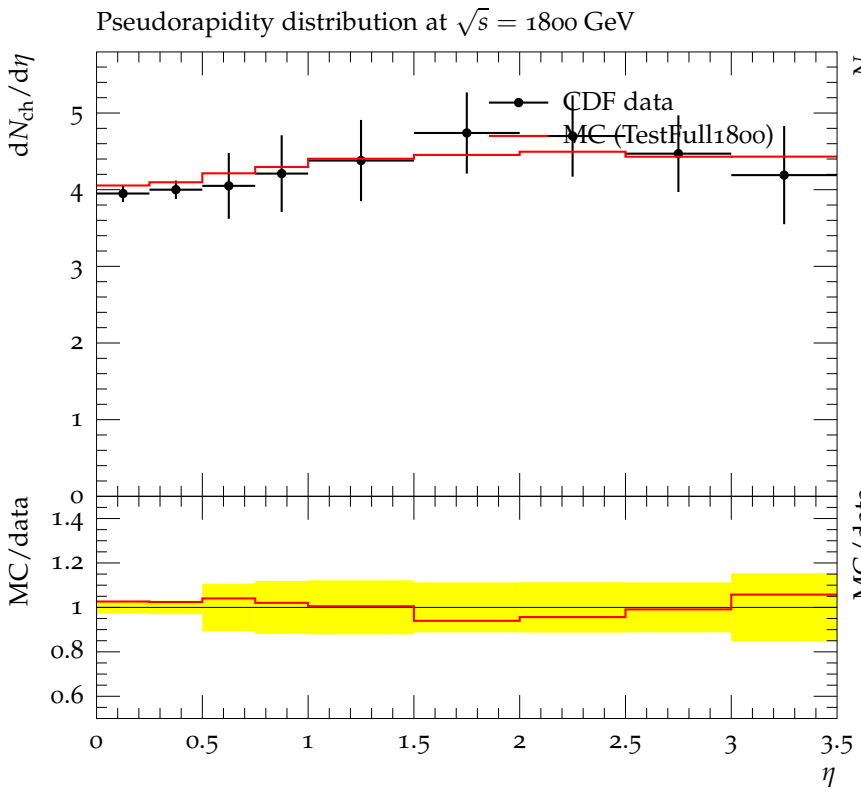
- Lund Dipole Model `arXiv:1004.5502` (Good–Walker)
  - $\alpha(0) = 1.21$, $\alpha' = 0.2 GeV^{-1}$
- Ryskin *et al.* `arXiv:0812.2407` (Three component pomeron)
  - $\alpha(0) = 1.3$, $\alpha' \leq 0.05 GeV^{-1}$
- Ostapchenko `arXiv:1003.0196` (Soft and hard pomeron)
  - $\alpha_h(0) = 1.35$, $\alpha'_h = 0.08 GeV^{-1}$
  - $\alpha_s(0) = 1.15$, $\alpha'_s = 0.14 GeV^{-1}$
- Gotsman *et al.* `arXiv:0805.2799` (Single pomeron)
  - $\alpha(0) = 1.335$, $\alpha' = 0.01 GeV^{-1}$
- Kaidalov *et al.* `arXiv:0910.2050` (Single pomeron)
  - $\alpha(0) = 1.12$, $\alpha' = 0.22 GeV^{-1}$
- Goulianos `arXiv:hep-ph/9502356` (Renormalised pomeron flux)
  - $\alpha(0) = 1.11$, $\alpha' = 0.26 GeV^{-1}$

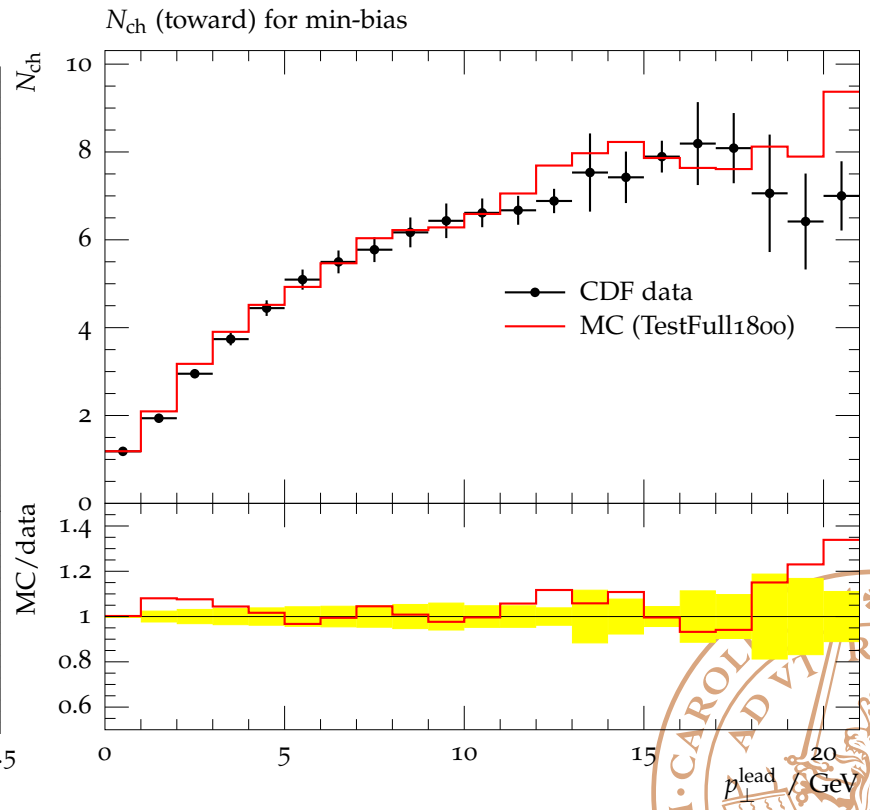
Summary

- Good–Walker can normally only describe low mass excitations.
- The Lund Dipole Cascade model can extend the Good–Walker formalism to large masses for $\gamma^* p$ and pp.
- Fluctuations, and thus $\sigma_{\text{diff ex}}$, are strongly suppressed by unitarity in pp.
- Diffractive excitation in pp is an expanding ring in b.
- DIPSY without saturation agrees very well with Tripple-Regge for a bare pomeron.
- corresponds to simple pole with $\alpha(0) = 1.21$ and $\alpha' = 0.2$. (With saturation cross section grows $\sim s^{0.1}$ up to 50 TeV.)
- Produces exclusive final states as well (soon to be published).

CDF

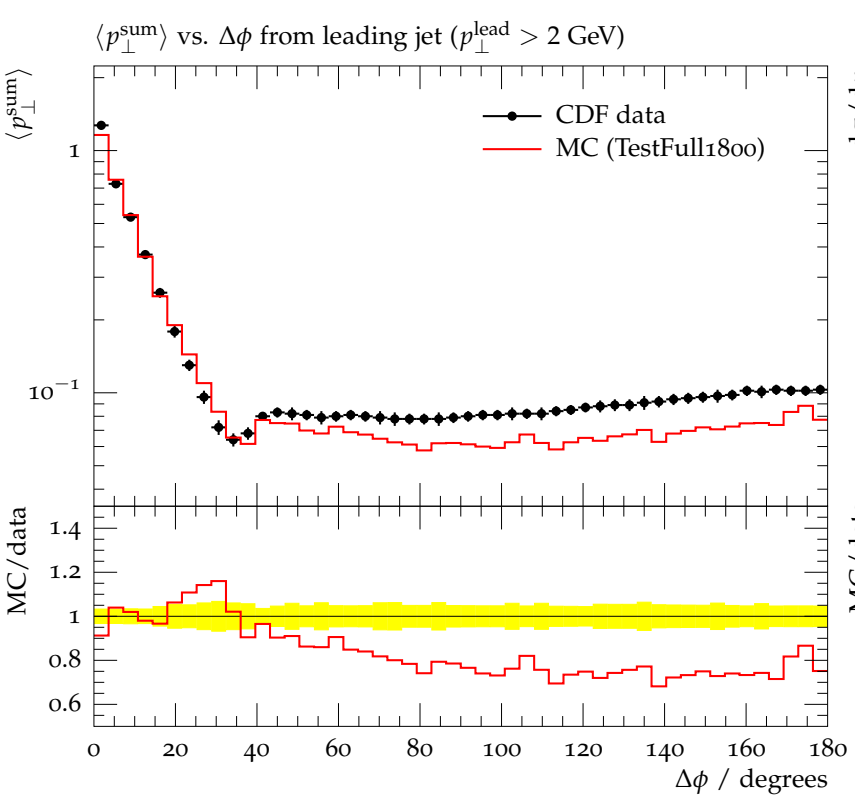
Pseudorapidity distribution and N_{ch} in towards region.

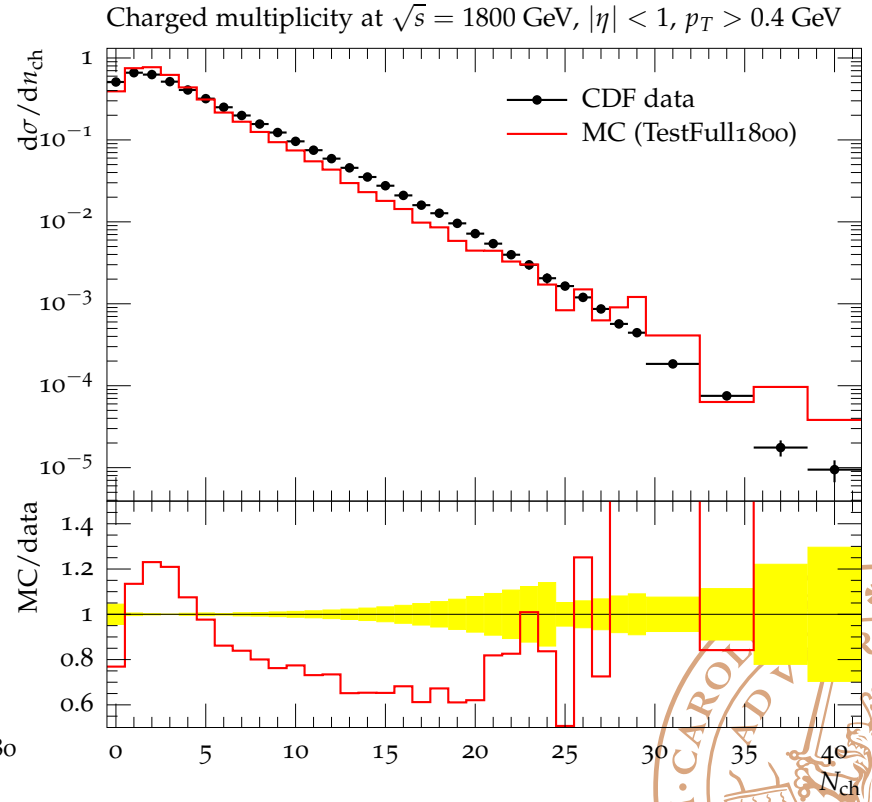




CDF

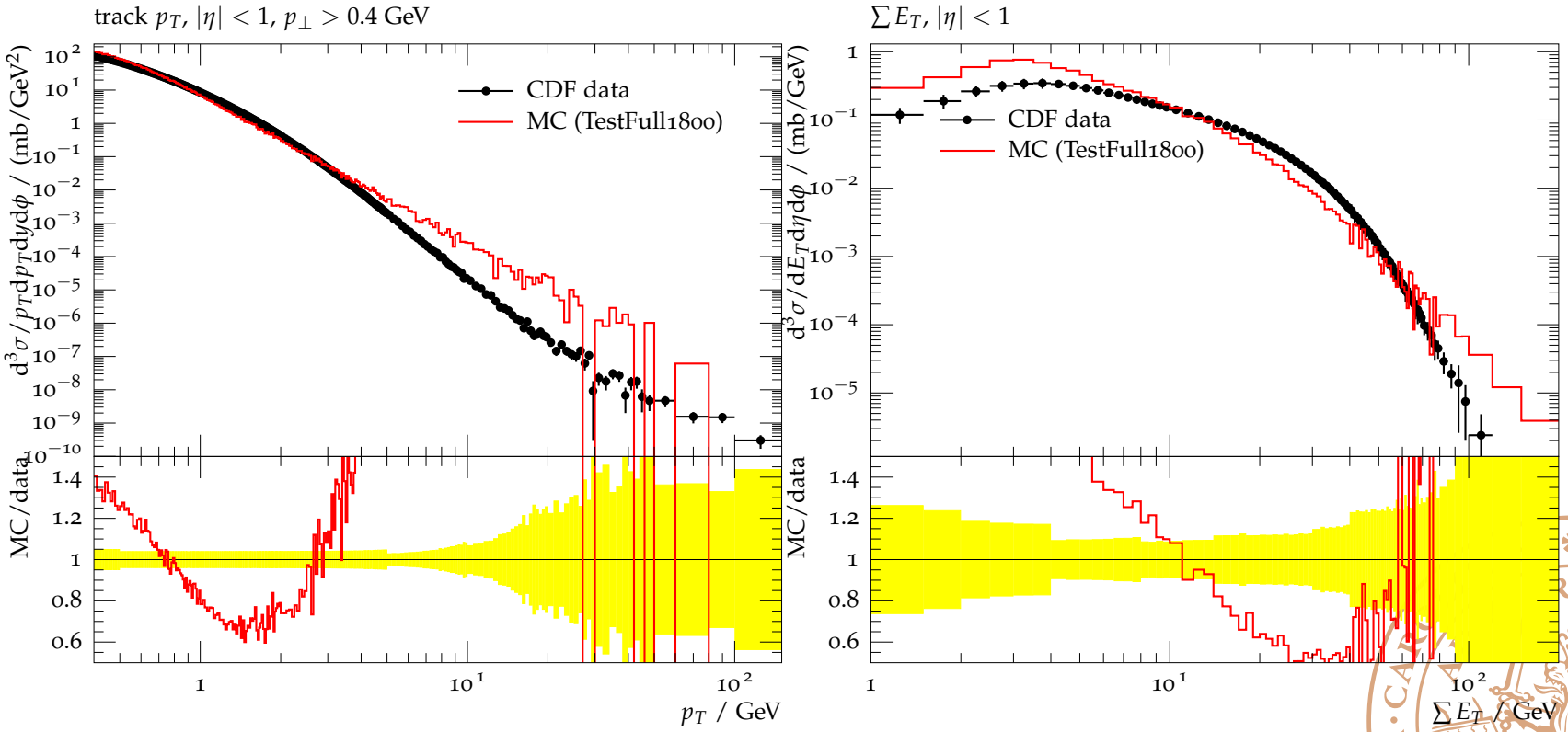
Angular distribution and multiplicity frequency.





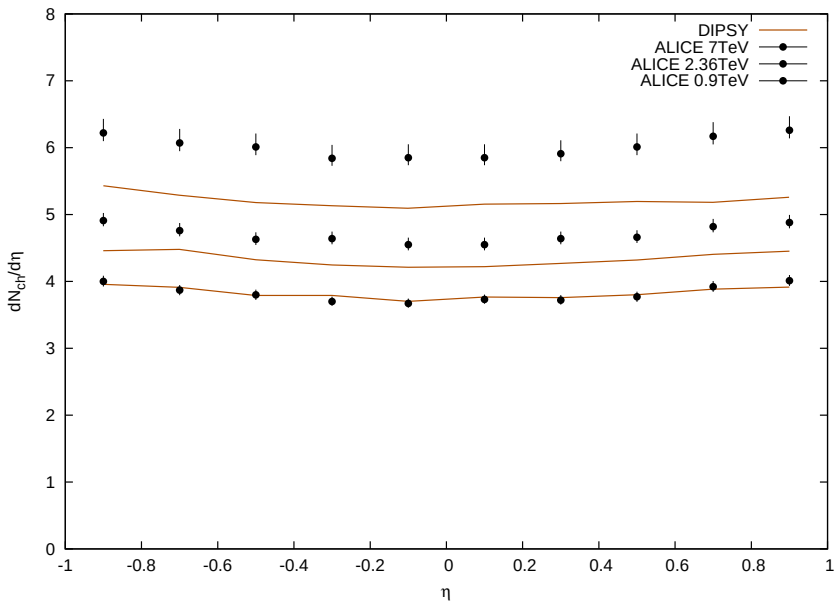
CDF

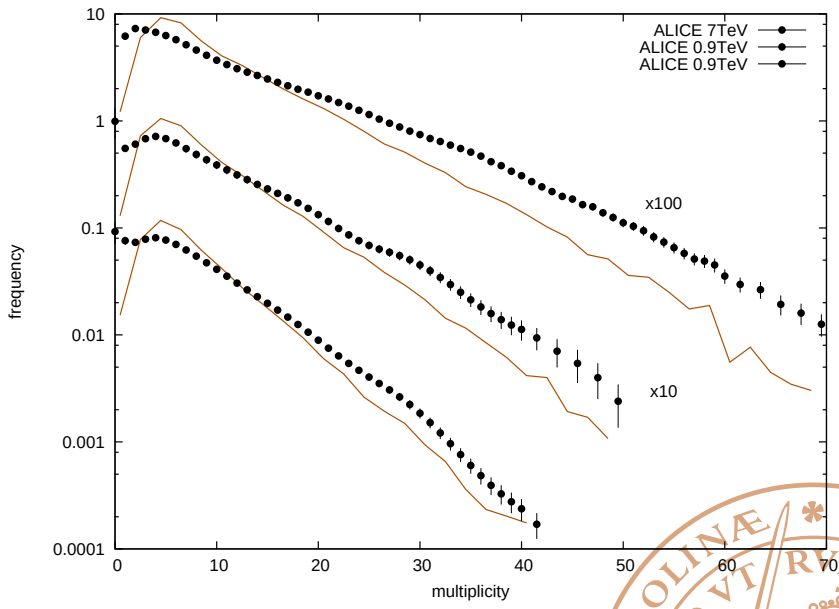
Track p_T and $\sum E_T$ distributions.



ALICE

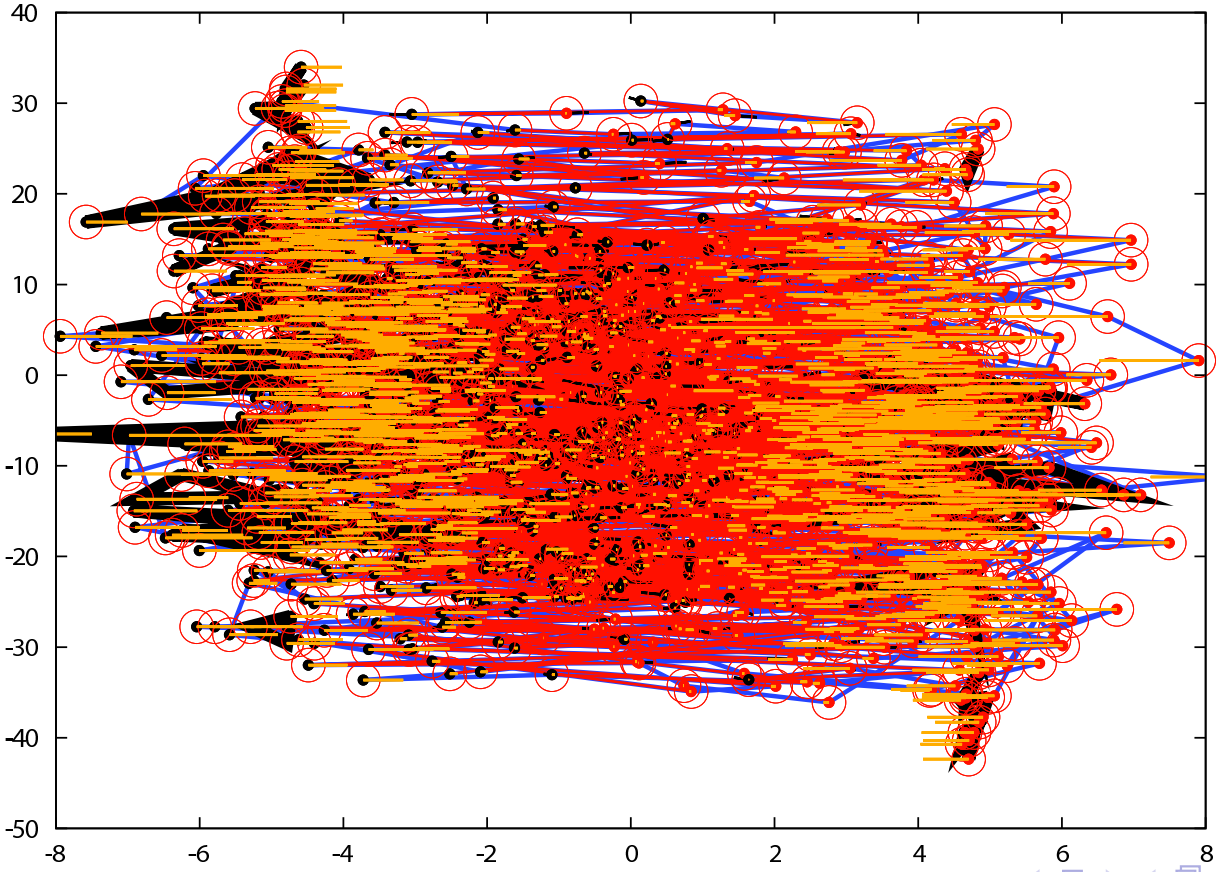
Rapidity distribution and Multiplicity frequency.





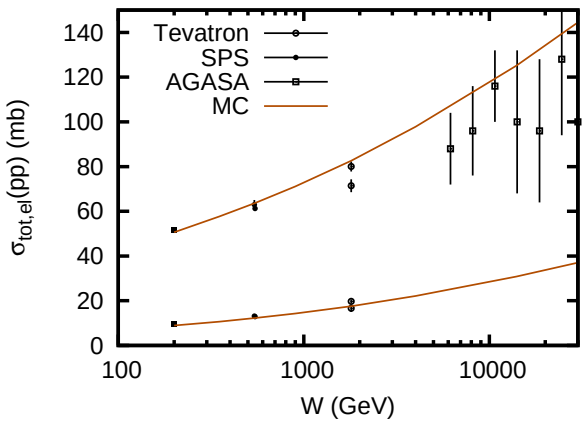
Heavy Ions

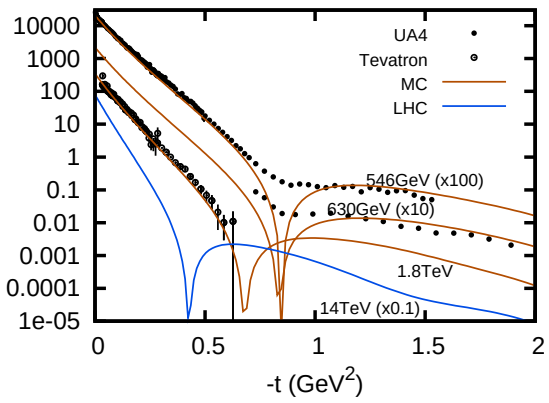
Central Au-Au collision in y-x_T space.

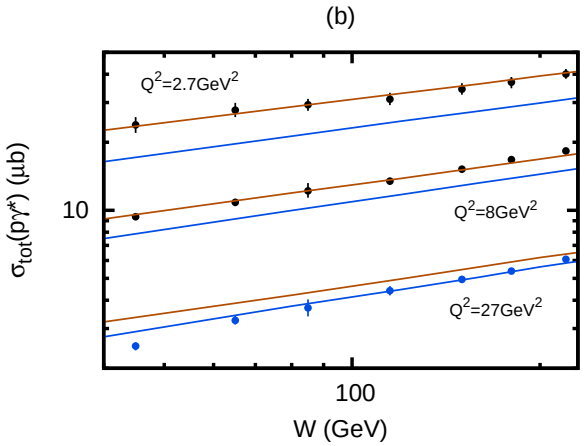


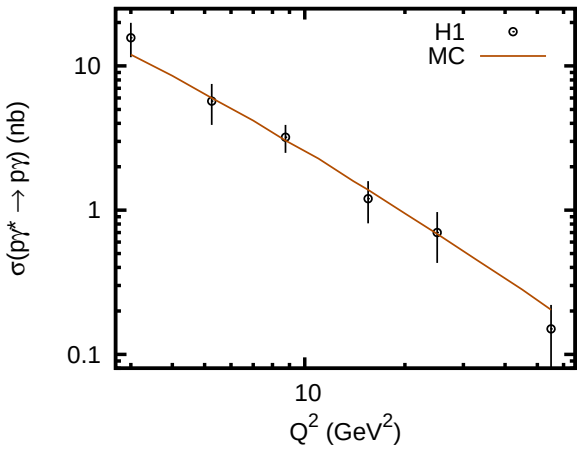
Some sample results

pp and $\gamma^\star$p: total, elastic and diffractive cross section.





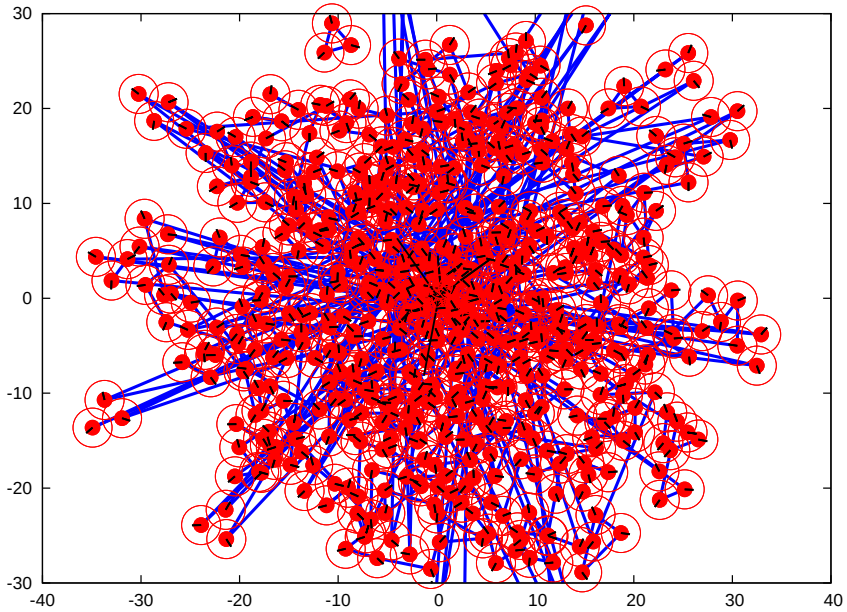


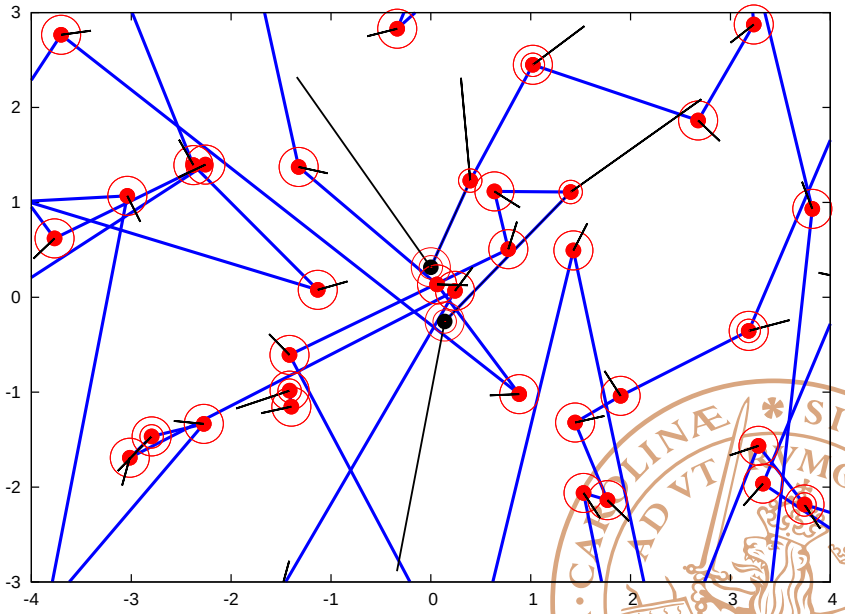


e^-A

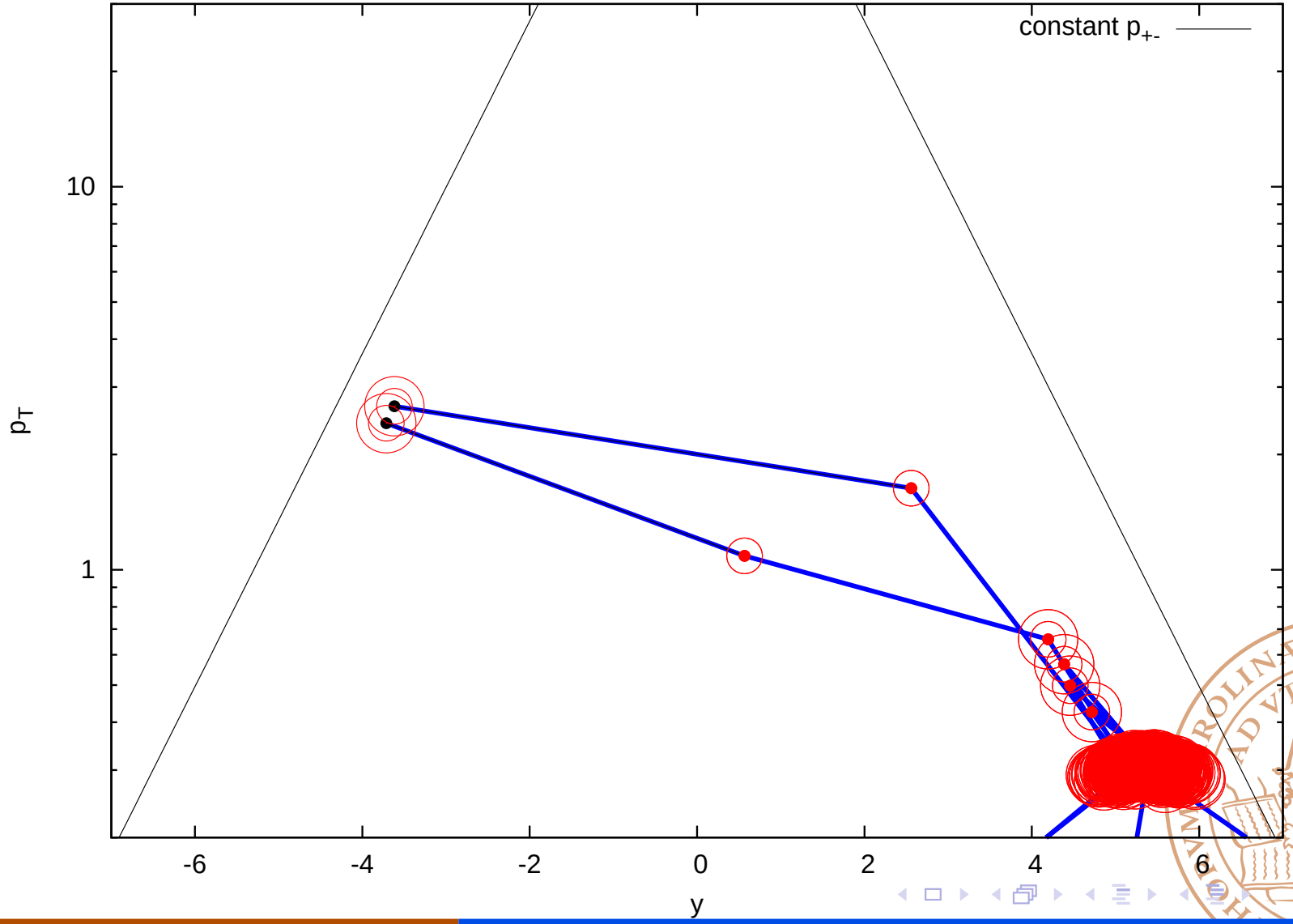
Collide a single dipole with a cascade of nucleons. Can again use same tools as for *pp*.

Here follows a sample event just as proof of concept:





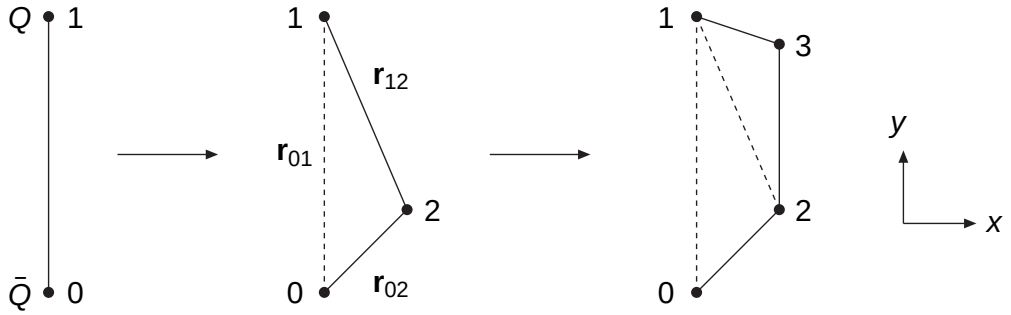
Or better seen in a y-p_T plot:



Evolution in rapidity

A colour dipole emits a gluon in transverse space with probability

$$\frac{\mathrm{d}P}{\mathrm{d}y} = \frac{\bar{\alpha}}{2\pi}\mathrm{d}^2 r_2 \frac{r_{01}^2}{r_{02}^2 r_{12}^2}$$



Equivalent to LL BFKL.

Interaction

A Born level calculation gives the collision amplitude for a pair of dipoles from different states:

$$f_{ij} = \frac{\alpha_s^2}{2} \ln^2 \left(\frac{r_{13} r_{24}}{r_{14} r_{23}} \right).$$

With the eikonal approximation, the total unitarised probability then becomes

$$t \equiv 1 - e^{-\sum f_{ij}}.$$

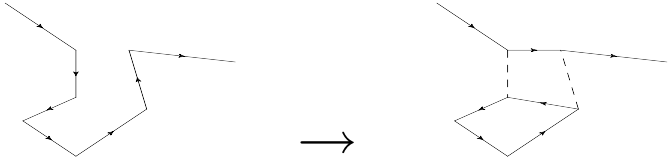
Modifications in DIPSY

Energy conservation

- Keep track of p_μ for all partons.
- Small dipoles $\leftrightarrow$ high p_T.
- Gives dynamic cutoff for small emissions.

Non-linear $2 \to 2$ swing:

- Saturation in evolution.



Confinement

- Supression of too large dipoles.