Up to now no complete model (Monte Carlo) including all facets --- elastic scattering, diffractive events, hard jets, etc. --- on the same footing. Important for the LHC

We seek a model that not only describes pure soft HE low $k_t$ data, (via Pomeron exchange and Reggeon FT), but which also extends into the large $k_t$ pQCD domain

To do this we need to introduce the partonic structure of the Pomeron: “soft” $\leftrightarrow$ “hard” Pomeron
Present data from the LHC

The $<p_T>$ of hadrons measured by ATLAS, CMS, ALICE is smaller than that expected from the **DGLAP-based** MC’s (which have strong-ordering in $k_T$ going from the protons to the central region).

Even after tuning the MC’s, the data have smaller $<p_T>$ and give a larger particle density $dN/dy$.

This indicates the need for a **BFKL-based** MC (with multi-Pomeron absorptive corrections), where we have diffusion in log $k_T$ and a growth of particle density as we go to large initial energy, that is smaller $x$. 
domain relevant to present LHC data

BFKL with multi-Pom corrections

\[ \sum (\alpha_s \ln 1/x)^n \]

\[ \ln \frac{1}{x} \]

\[ \ln Q^2 \]
Analyses of “soft” HE data using multi-Pom. exch. framework

1986: Kaidalov, Ponomarev, Ter-M (enh. single-ch eikonal)

History of KMR analyses:
2000 KMR: similar model with 2-ch eikonal (+Tevatron data, + study of $B_{el}(s,t)$)
2007 KMR: extended to give partonic interpretation via evolution eqs. in rapidity with absorptive effects
2008 KMR: Pomeron cut mimicked by 3 poles (matrix form of evol. eqs.) – diffusion in $b$ and some in $\log k_T$
2010 KMR: $k_T$ dependence of Pomeron explicit in integral form of evol. eqs. (diffusion in $\log k_T$) $\Rightarrow$ “predicts” PDFs and diffractive PDFs (constraint). Can form the basis of BFKL-motivated “hard-soft” Monte Carlo.

see also Ostapchenko, GLMM, Poghosyan and Kaidalov, Flensburg and Gustafsson
Model for “soft” high-energy interactions

needed to ---- understand asymptotics, intrinsic interest
----- describe “min. bias/underlying” events
----- calc. rap. gap survival prob. $S^2$ for diff. processes
----- devise “all-purpose” Monte Carlo

Model should:

1. be self-consistent theoretically --- satisfy unitarity
   → importance of absorptive corrections
   → importance of multi-Pomeron interactions

2. agree with soft data (-t<0.5 GeV$^2$) in CERN-ISR to Tevatron range
   $$\sigma_{\text{tot}}, \frac{d\sigma_{\text{el}}}{dt}, \frac{d\sigma_{\text{SD}}}{dt dM^2}(pp \rightarrow pX)$$ etc.

3. include $b$ and log $kT$ dependence of the Pomeron to study effects of absorption

4. be suitable for “all-purpose” MC --- partonic approach
Low-mass diffractive dissociation introduce diverse estates $\phi_i, \phi_k$ (combinations of $p, p^*, ..$) which only undergo “elastic” scattering (Good-Walker)

\[ \text{Im } T_{ik} = \prod_{k}^{i} = 1 - e^{-\Omega_{ik}/2} = \sum \prod_{n=1}^{\infty} \Omega_{ik} \]

include high-mass diffractive dissociation

\[ \Omega_{ik} = \prod_{k}^{i} + \prod_{k}^{i} \{ M \} + \prod_{k}^{i} + \cdots + \prod_{k}^{i} + \cdots \]
DL parametrization:

$$\alpha_P(t) = 1.08 + 0.25t$$

KMR parametrization includes absorption via multi-Pomeron effects

Absorption crucial at small $b$
A vacuum-exchange object drives soft HE interactions. Not a simple pole, but an enigmatic non-local object. Rising $\sigma_{\text{tot}}$ means multi-Pom diags (with Regge cuts) are necessary to restore unitarity. $\sigma_{\text{tot}}$, $d\sigma_{\text{el}}/dt$ data, described, in a limited energy range, by eff. pole $\alpha_P^{\text{eff}} = 1.08 + 0.25t$

Sum of ladders of Reggeized gluons with, in LLx BFKL, a singularity which is a cut and not a pole (or with running $\alpha_s$ a series of poles). When HO are included the intercept of the BFKL/hard Pomeron is $\Delta = \alpha_P(0)-1 \sim 0.3$

We argue that there exists only one Pomeron, which makes a smooth transition between the hard and soft regimes.
Evidence that the soft Pomeron in soft domain has qualitatively similar structure to the hard Pomeron

No irregularity in HERA data in the transition region $Q^2 \sim 0.3 – 2 \text{ GeV}^2$. Data are smooth through this region.

Small slope $\alpha' < 0.05 \text{ GeV}^{-2}$ of bare Pomeron trajectory is found in global analyses of “soft” data after accounting for absorptive corrections and secondary Reggeons. So typical values of $k_T$ inside Pomeron amplitude are relatively large ($\alpha' \sim 1/k_T^2$).

These global analyses of “soft” data find bare Pomeron intercept $\Delta = \alpha_p(0) - 1 \sim 0.3$ close to the intercept of the hard/pQCD Pomeron after NLL corrections are resummed.
The data on vector meson electro-production at HERA imply a power-like behaviour which smoothly interpolates between the “effective” soft value $\sim 1.1$ at $Q^2\sim 0$, and a hard value $\sim 1.3$ at large $Q^2$.

In summary, the bare pQCD Pomeron amplitude, with trajectory $\alpha_P \sim 1.3 + 0 \, t$, is subject to increasing absorptive effects as we go to smaller $k_T$ which allow it to smoothly match on to the attributes of the soft Pomeron.

In the limited energy interval up to the Tevatron energy, some of these attributes (specifically those related to the elastic amplitude) can be mimicked by an effective Pomeron pole with trajectory $\alpha_P^\text{eff} = 1.08 + 0.25 \, t$. 
Vector meson production at HERA

Hard energy dependences:
- $\alpha_{P}(0) \approx 1.3$
- $\alpha_{P}(0) \approx 1.1$

Soft and hard production:
- $Q^2 + M^2$ vs. $\alpha_{P}(0)$
A partonic approach to soft interactions

We have seen that it is reasonable to assume that in the soft domain the soft Pomeron has the general properties expected from the hard/QCD Pomeron; at least there is a smooth transition from the soft to hard Pomeron.

This opens the way to extend the description of HE “soft” interactions to the perturbative very low $x, k_T \sim$few GeV domain, a region relevant to the LHC.

We start from the partonic ladder structure of the Pomeron, generated by the BFKL-like evolution in rapidity.
Partonic structure of Pomeron

"bare" Pomeron pole

\[ \Omega = \Omega_{ik}(y, k_t, b) \]

\[ \frac{\partial \Omega(y, k_t)}{\partial y} = \tilde{\alpha}_s \int d^2 k_{t'} K(k_t, k_{t'}) \Omega(y, k') \]

\[ \tilde{\alpha}_s \equiv \frac{3\alpha_s}{\pi} \]

k\_t dependence now included in integral form

BFKL-like kernel

Naïve:

\[ \frac{\partial \Omega}{\partial y} = \Delta \Omega \quad \text{where} \quad \Delta = \tilde{\alpha}_s \langle K \rangle \]

\[ \Omega = e^{\Delta y} = x^{-\Delta} \sim s^{\alpha(0)-1} \quad (\text{so} \quad \Delta = \alpha(0) - 1) \]

BFKL:

LL(1/x):

\[ \Delta = 4\ln 2 \tilde{\alpha}_S \quad \rightarrow \quad \Delta \sim 0.5 \]

NLL(1/x):

\[ \Delta = 4\ln 2 \tilde{\alpha}_S(1 - 6\tilde{\alpha}_S) : \quad \text{resum all major HO} \rightarrow \quad \Delta \sim 0.3 \]

Evolution in rapidity:

\[ \frac{\partial \Omega(y, k_t)}{\partial y} = \tilde{\alpha}_s \int d^2 k_{t'} K(k_t, k_{t'}) \Omega(y, k_{t'}) \]

generates ladder
evolution in rapidity, \( \frac{\partial \Omega(y, k_t)}{\partial y} = \tilde{\alpha}_s \int d^2 k_t' K(k_t, k_t') \Omega(y, k_t') \), generates ladder

\[
\begin{align*}
\Omega & = \underbrace{\quad + \quad}_{\Omega_{ik}(y=0)} \\
\text{At each step } k_t \text{ and } b \text{ of parton can be changed – so, in principle, we have 2-variable integro-differential eq. to solve}
\end{align*}
\]

\[
\Omega \Rightarrow \quad = \quad
\]

- We use a simplified form of the kernel \( K \) which incorporates the main features of BFKL – diffusion in \( \log k_t^2 \), \( \Delta \sim 0.3 \)

- \( b \) dependence during the evolution is prop’ to the Pomeron slope \( \alpha' \), which is v.small (\( \alpha' < 0.05 \text{ GeV}^{-2} \)) -- so ignore. Only \( b \) dependence comes from the starting evol\textsuperscript{n} distrib\textsuperscript{n}

- Evolution gives\[ \Omega = \Omega_{ik}(y, k_t, b) \]
Multi-Pomeron contributions

- e.g. triple-Pomeron diagram: parton $c$ has extra scattering with "target" $k$

- Many rescatt: different no. of ladders between $c$ and $k$

\[
\frac{\partial \Omega_k(y)}{\partial y} = \bar{\alpha}_s \int d^2 k'_t \, e^{-\lambda \Omega_k(y)/2} \, K(k_t, k'_t) \, \Omega_k(y)
\]

where $\lambda \Omega_k$ reflects the different opacity of "target" $k$ felt by $c$, rather opacity $\Omega_k$ than felt by "beam" $i$ \[\lambda \approx 0.25\]
Multi-Pomeron contributions continued

so far

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\]

then we include a full set of fan diagrams for rescatt of c with k

\[
e^{-\lambda \Omega_k(y)/2}
\]

Now include rescatt of c with “beam” i

\[
\begin{align*}
\frac{\partial \Omega_k(y)}{\partial y} &= \bar{\alpha}_s \int d^2k'_t \exp(-\lambda(\Omega_k(y) + \Omega_i(y'))/2) K(k_t, k'_t) \Omega_k(y) \\
\frac{\partial \Omega_i(y')}{\partial y'} &= \bar{\alpha}_s \int d^2k'_t \exp(-\lambda(\Omega_i(y') + \Omega_k(y))/2) K(k_t, k'_t) \Omega_i(y')
\end{align*}
\]

solve iteratively for \(\Omega_{ik}(y, k_t, b)\)
Aim is to study main features of data in terms of a realistic model with just a few physically motivated parameters:

- \( \Delta \) and \( d \) which specify the simplified BFKL kernel
d gives diffusion in \( \log k_t^2 \), \( \Delta \) controls \( s \) dependence

- \( \beta_0 \) specifies the Pomeron-proton coupling

- \( c_1 \) and \( c_2 \) specify proton form factor

- \( \lambda \) determines multi-Pomeron couplings, which are constrained by data on high-mass diffractive dissociation

- \( \gamma \) specifies (Good-Walker) diffractive eigenstates, which is constrained by data on low-mass diffractive dissociation

Good global fit to soft data achieved
Can predict PDFs and diffractive PDFs at small \( x, Q^2 \) (underway)
Can form basis of all-purpose MC – prelim studies encouraging Krauss, Hoeth, Zapp
Observables from $\Omega_{ik}(k_t, b, y)$

example 1: total cross section

$$\Omega_{ik}^{\text{eff}}(\vec{b}, Y) = \int d^2 k_t \int \Omega_i(\vec{k}_t, \vec{b}_1, \vec{b}_2, y) \Omega_k(\vec{k}_t, \vec{b}_1, \vec{b}_2, Y - y) \, d^2 b_1 d^2 b_2 \delta^{(2)}(\vec{b}_1 - \vec{b}_2 - \vec{b})$$

$$\sigma_{\text{tot}} = 2 \sum_{i,k} |a_i|^2 |a_k|^2 \int (1 - e^{-\Omega^{\text{eff}}/2}) \, d^2 b$$

Diagram: 
- $\vec{i}$ to $Y$ with $b_1$.
- $k$ to $0$ with $b_2$.
- $\Omega_{i(k)}$.
- $\Omega_{k(i)}$.
- $\vec{b}_1$ to $c$ with $\vec{b}$.
- $\vec{b}_2$ to $k$ with $\vec{b}$. 

rescatt. between low $x$, low $k_t$ partons and “target” distorts input distribn -- violates soft-hard factorizn

\[ S^2_{ik}(\vec{b}) = \exp(-\Omega_{ik}^{\text{eff}}(\vec{b})) \]

large $k_t$ partons -- rescatt. negligible

Extra suppression due to enhanced rescattering estimated to be small for Higgs production, but is predicted to be significant (~1/3) for central exclusive production of much less massive systems, for example $\chi$ production at the LHC
Global fit to soft data:
total, elastic, low- and high-mass diffraction

cross sections in mb

<table>
<thead>
<tr>
<th></th>
<th>Tevatron</th>
<th>14 TeV LHC</th>
<th>$\sqrt{s} = 10^5$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{tot}$</td>
<td>74.0 (73.9)</td>
<td>$\sigma_{tot}$ 88.0 (86.3)</td>
<td>98.0 (94.3)</td>
</tr>
<tr>
<td>$\sigma_{el}$</td>
<td>16.3 (15.1)</td>
<td>$\sigma_{el}$ 20.1 (18.1)</td>
<td>22.9 (20.0)</td>
</tr>
<tr>
<td>$\sigma_{SD}$</td>
<td>10.9 (12.7)</td>
<td>$\sigma_{SD}$ 13.3 (16.1)</td>
<td>15.7 (17.7)</td>
</tr>
<tr>
<td>$\sigma_{SD}^{low M}$</td>
<td>4.3 (6.0)</td>
<td>5.1 (7.0)</td>
<td>5.7 (7.9)</td>
</tr>
<tr>
<td>$\sigma_{SD}^{high M}$</td>
<td>6.5 (6.7)</td>
<td>8.1 (9.1)</td>
<td>10.0 (9.8)</td>
</tr>
<tr>
<td>$\sigma_{DD}$</td>
<td>7.2 (8.7)</td>
<td>$\sigma_{DD}$ 13.4 (12.9)</td>
<td>17.3 (21.1)</td>
</tr>
<tr>
<td>$\sigma_{DD}^{low M}$</td>
<td>0.2 (0.5)</td>
<td>0.2 (0.5)</td>
<td>0.2 (0.6)</td>
</tr>
<tr>
<td>$\sigma_{DD}^{high M}$</td>
<td>4.5 (4.0)</td>
<td>9.3 (5.9)</td>
<td>11.7 (12.9)</td>
</tr>
<tr>
<td>$\sigma_{DD}^{(high M*low M)}$</td>
<td>2.1 (3.6)</td>
<td>2.9 (5.2)</td>
<td>3.8 (6.0)</td>
</tr>
<tr>
<td>$\sigma_{DD}^{(SD*SD)}$</td>
<td>0.4 (0.7)</td>
<td>1.0 (1.3)</td>
<td>1.6 (1.6)</td>
</tr>
</tbody>
</table>

example: 2007 fit; the 2010 fit is still preliminary
The multi-Pomeron couplings are not known.

The $m \rightarrow n$ Pomeron couplings implied by the evolution equations, which appear to be the most physically natural choice, are

$$g(m \rightarrow n) = nm \lambda^{n+m-2} g_N.$$  

We are exploring the effect of models without the $nm$ factor, which result in less absorption, giving for example $\sigma_{\text{total}} \sim 100$ mb at 14 TeV.
Conclusions

-- Obtained a fully consistent description of high-energy soft interactions; new model based on both a multi-channel eikonal and multi-Pomeron interactions (with $k_T$ dependence)

-- screening/unitarity/absorptive corrections are appreciable at LHC energies. \( \sigma_{\text{total}} \) (14 TeV) \( \sim \) 90 - 100 mb

-- soft-hard Pomeron transition emerges
  “soft” compt. --- heavily screened --- little growth with $s$
  “hard” compt. --- little screening --- large growth (~pQCD)

-- Model predicts PDFs and diffractive PDFs (gives constraint!)

-- Model allows both eikonal and enhanced rescatt. corrections, so gap survival prob. can be calculated for any diffractive proc.

-- Model has partonic interpretation, so can form basis of “all purpose” Monte Carlo. Preliminary results encouraging - Krauss, Hoeth, Zapp