Geometric Scaling

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First Observation

- original study
  - $\sigma^{\gamma^*p}(Q^2, x) = \sigma^{\gamma^*p}(\tau)$ where $\tau = Q^2 \times (x/x_0)^{\lambda}$
stochastic extension of Balitsky-Kovchegov equation for dipole amplitude

\[ \frac{\partial T}{\partial Y} = \alpha_s \left[ \chi(-\partial L) T - T^2 \right] \]

- \( \chi \) is the BFKL kernel, \( L = \log Q^2 \), \( Y = \log \frac{1}{x} \)
- BFKL equation when \( \alpha_s \) constant and \( T^2 \) neglected

- \( \alpha_s \) constant
  - solution does not depend independently on \( Y \) and \( \log Q^2 \) but a combination of both \( \rightarrow \) scaling
  - Fixed Coupling: \( \tau = L - \lambda Y \)
stochastic extension of Balitsky-Kovchegov equation

\[ \frac{\partial T}{\partial Y} = \alpha_s(Q^2) \left[ \chi(-\partial_L) T - T^2 \right] \]

\( \alpha_s \) running

- \( \alpha_s \sim 1/\log Q^2 \)
- \( T^2 \) is expected to follow the scaling
- but both \( L \partial T / \partial Y \) and \( \chi(-\partial_L) \) cannot follow the same scaling

<table>
<thead>
<tr>
<th>scaling</th>
<th>( \partial_L T )</th>
<th>( L \partial T / \partial Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T(L - \lambda \sqrt{Y}) )</td>
<td>( \frac{\partial T}{\partial L}(L - \lambda \sqrt{Y}) = \frac{\partial T}{\partial L} ) scaling</td>
<td>( L \frac{\partial T}{\partial Y} = \frac{\lambda L}{2\sqrt{Y}} T ) approx. scaling</td>
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<td>( T(L - \lambda Y/L) )</td>
<td>( \frac{\partial T}{\partial L}(L - \lambda Y/L) = \frac{\lambda Y}{L^2} \frac{\partial T}{\partial L} ) approx. scaling</td>
<td>( L \frac{\partial T}{\partial Y}(L - \lambda Y/L) = -\lambda \frac{\partial T}{\partial Y} ) scaling</td>
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stochastic extension of Balitsky-Kovchegov equation

\[ \frac{\partial T}{\partial Y} = \alpha_s(Q^2) \left[ \chi(-\partial L) T - T^2 \right] \]

\( \alpha_s \) running

\( \alpha_s \sim 1/\log Q^2 \)

\( T^2 \) is expected to follow the scaling

but both \( L \partial T / \partial Y \) and \( \chi(-\partial L) \) cannot follow the same scaling

two solutions:

- Running Coupling I: \( \tau = L - \lambda \sqrt{Y} \)
- Running Coupling II: \( \tau = L - \lambda Y/L \) (see G. Beuf, arXiv:0803.2167)
Scalings in DIS

- extended Balitsky-Kovchegov equation

\[
\frac{\partial T}{\partial Y} = \alpha_S \left[ \chi(-\partial L)T - T^2 + \sqrt{\alpha_S^2 \kappa T} \nu(L, Y) \right]
\]

- \(\alpha_S\) constant, \(\nu\)
  - \(\nu\) is a gaussian noise corresponding to the fluctuation of number of gluons
  - corresponds to pomeron loops (gluon splitting)
  - Diffusive Scaling: \((L - \lambda Y)/\sqrt{Y}\)
Quality Factor Method

- test different scaling laws $\tau = \tau(Q^2, x; \lambda)$ on data $\sigma \sim F_2/Q^2$
- quality factor method
  - normalise data sets $v_i = \log(\sigma_i)$ and scaling laws $u_i = \tau_i(\lambda)$ between 0 and 1
  - order in $u_i$
  - define quality factor

$$QF(\lambda) = \left[ \sum_i \frac{(v_i - v_{i-1})^2}{(u_i - u_{i-1})^2 + \epsilon^2} \right]^{-1}$$

- fit $\lambda$ to maximase $QF$

Scaling Tests in DIS

- combined $F_2$ measurements from H1/ZEUS
- stay in perturbative domain: $4 \leq Q^2 \leq 150$ GeV$^2$
- avoid region where valence quarks dominate: $x \leq 10^{-2}$
- avoid high $y$ region where $F_L$ contributes: $y \leq 0.6$
- 117 data points
Quality Factor

- comparison of $1/QF$ for FC, RCI, RCII, DS
Comparison of Different Scalings

- value of parameters and QF for $Q^2 \geq 4 \text{ GeV}^2$

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<thead>
<tr>
<th>scaling</th>
<th>$\lambda$</th>
<th>$1/QF$</th>
<th>$\tau = \log Q^2 - \lambda \log(\frac{1}{x})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Coupling</td>
<td>0.31</td>
<td>150.2</td>
<td>$\tau = \log Q^2 - \lambda \log(\frac{1}{x})$</td>
</tr>
<tr>
<td>Running Coupling I</td>
<td>1.61</td>
<td>137.9</td>
<td>$\tau = \log Q^2 - \lambda \sqrt{\log(\frac{1}{x})}$</td>
</tr>
<tr>
<td>Running Coupling II</td>
<td>2.76</td>
<td>124.3</td>
<td>$\tau = \log(Q^2/0.2^2) - \lambda \frac{\log(\frac{1}{x})}{\log(Q^2/0.2^2)}$</td>
</tr>
<tr>
<td>Diffusive Scaling</td>
<td>0.31</td>
<td>210.7</td>
<td>$\tau = \frac{\log Q^2}{\sqrt{\log \frac{1}{x}}} - \lambda \log(\frac{1}{x})$</td>
</tr>
</tbody>
</table>

- FC, RCI and RCII favoured, DS disfavoured
- no significant improvement with additional parameters ($Q_0, Y_0$)
- results compatible with our previous study using older data (arXiv:0803.2186)
Running Coupling I: \[ \tau = \log \frac{Q^2}{Q_0^2} - \lambda \log \left( \frac{1}{x} \right) \]
dilute regime (no saturation) \( \rightarrow \tau > 0 \)
\( \tau \) can be shifted by changing \( Q_0 \)
Fits to HERA Data

- fit to HERA data inspired by RCI

\[
\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \sqrt{\log\left(\frac{1}{x}\right) - Y_0}
\]

\[
\sigma = N \exp\left(-\alpha \tau\right) \exp\left(\frac{-\beta \tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)
\]

- fit formula deduced from the dipole amplitude with saturation (Gregory Soyez) with asymptotic expression of the Airy function which is a solution of Balitsky-Kovchegov equation

- 6 fit parameters: \(\lambda, \alpha, \beta, Q_0, Y_0, N\)

- explicit moderate scaling violation: \((\log 1/x - Y_0)^{1/4}\)
  - fits performed with and without the scaling violation term (predicted by the dipole model)

- \(\tau\) must be positive in the dilute regime
Fit Results

- fit variables:

\[
\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \sqrt{\log\left(\frac{1}{x}\right) - Y_0}
\]

\[
\sigma = N \exp(-\alpha \tau) \exp\left(\frac{-\beta \tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)
\]

- Fit I: \(\chi^2 = 130.1\) for 117 points (\(\chi^2/dof = 1.2\))
- Fit II (without the scaling violation term): \(\chi^2 = 119.0\)

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<th>Fit II</th>
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<tr>
<td>(\lambda)</td>
<td>1.54 ± 0.02</td>
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</tr>
<tr>
<td>(\alpha)</td>
<td>0.34 ± 0.01</td>
<td>0.18 ± 0.01</td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.24 ± 0.01</td>
<td>0.18 ± 0.01</td>
</tr>
<tr>
<td>(Q_0)</td>
<td>0.079 ± 0.01</td>
<td>0.064 ± 0.01</td>
</tr>
<tr>
<td>(Y_0)</td>
<td>-1.46 ± 0.02</td>
<td>0.50 ± 0.02</td>
</tr>
<tr>
<td>(N)</td>
<td>0.51 ± 0.01</td>
<td>0.72 ± 0.01</td>
</tr>
</tbody>
</table>
• good description of HERA data at low $Q^2$ and low $x$
• fit does not describe the turn-over of $\sigma_r$ at low $x$ (high $y$)

$$\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$$

• model of $F_L$ using RCI needed (in progress)
fair description of data at lower $Q^2$

parameterisation of $F_L$ needed in order to describe high $y$ data
Extrapolation to $Q^2$

- high $x$ not well described
- valence quarks needed
Other Fits

- similar formula for RCII:

\[
\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \frac{\log\left(\frac{1}{x}\right) - Y_0}{\log\left(\frac{Q^2}{Q_0^2}\right)}
\]

\[
\sigma = N \exp(-\alpha \tau) \exp\left(\frac{-\beta \tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)
\]

\[\chi^2 = 190.4 \rightarrow \text{worse than RCI}\]

- similar formula for FC:

\[
\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \log\left(\frac{1}{x}\right)
\]

\[
\sigma = N \exp(-\alpha \tau) \exp\left(\frac{-\beta \tau^2}{(\log 1/x - Y_0)}\right)
\]

\[\chi^2 = 156.4 \rightarrow \text{worse than RCI}\]

\[\chi^2 = 230.5 \text{ without the scaling violation term}\]
Conclusion

- different scalings studied in $F_2$ data: Fixed Coupling, Running Coupling I and II, Diffusive Scaling
- Fixed Coupling, Running Coupling I and II lead to a good description of data using the QF formalism
- Diffusive Scaling disfavoured

- fit of $F_2$ data using RCI (parameterised with or without moderate scaling violations) leads to a good description of data at low $Q^2$ and low $x$
- fits disfavour RCII and FC

- outlook:
  - fits of lower $Q^2$ data in the saturation region
  - fits of high $y$ data including $F_L$ parameterisation
  - comparison with numerical solution of BK equation with $\alpha_S$ running