Geometric Scaling

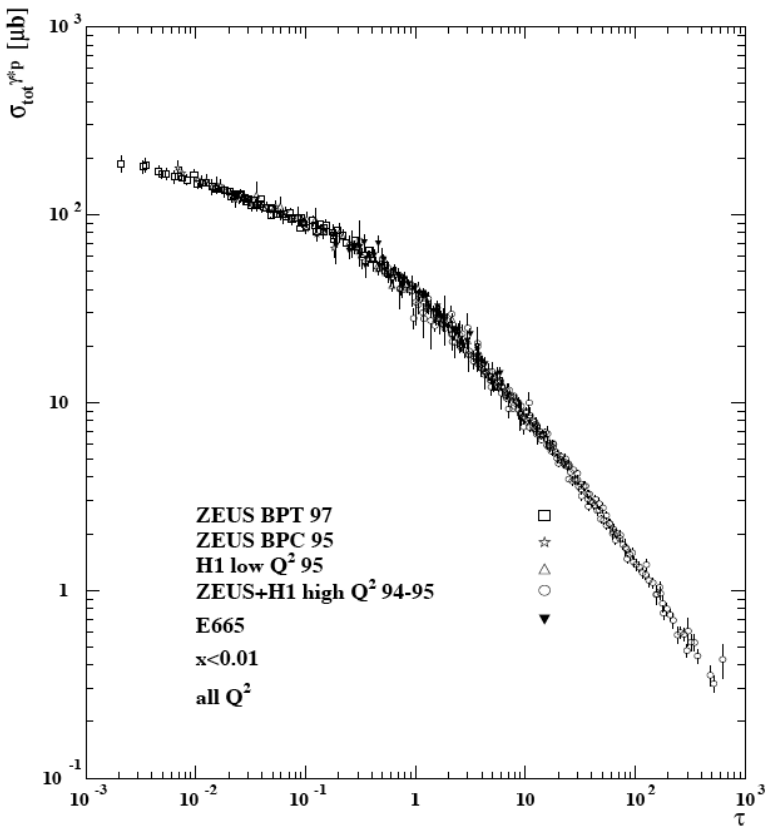
David Šálek

Charles University

September 11, 2010

First Observation

- original study
  - $\sigma^{\gamma^* p}(Q^2, x) = \sigma^{\gamma^* p}(\tau)$ where $\tau = Q^2 \times (x/x_0)^\lambda$
  - A. M. Stasto, K. Golec-Biernat, and J. Kwiecinski, Phys. Rev. Lett. 86 (2001) 596



Scalings in DIS

- stochastic extension of Balitsky-Kovchegov equation for dipole amplitude

$$\frac{\partial T}{\partial Y} = \alpha_S \left[\chi(-\partial_L) T - T^2\right]$$

  - χ is the BFKL kernel, $L = \log Q^2$, $Y = \log 1/x$
  - BFKL equation when α_S constant and T^2 neglected
- α_S constant
  - solution does not depend independently on Y and $\log Q^2$ but a combination of both $\rightarrow$ scaling
  - Fixed Coupling: $\tau = L - \lambda Y$

Scalings in DIS

- stochastic extension of Balitsky-Kovchegov equation

$$\frac{\partial T}{\partial Y} = \alpha_S(Q^2)\left[\chi(-\partial_L)T - T^2\right]$$

- α_S running
  - $\alpha_S \sim 1/logQ^2$
  - T^2 is expected to follow the scaling
  - but both $L\partial T/\partial Y$ and $\chi(-\partial_L)$ cannot follow the same scaling

scaling	$\partial_L T$	$L\partial T/\partial Y$
$T(L-\lambda\sqrt{Y})$	$\frac{\partial T}{\partial L}(L-\lambda\sqrt{Y}) = \frac{\partial T}{\partial L}$ scaling	$L\frac{\partial T}{\partial Y} = \frac{\lambda L}{2\sqrt{Y}}T$ approx. scaling
$T(L-\lambda Y/L)$	$\frac{\partial T}{\partial L}(L-\lambda Y/L) = \left(1+\frac{\lambda Y}{L^2}\right)\frac{\partial T}{\partial L}$ approx. scaling	$L\frac{\partial T}{\partial Y}(L-\lambda Y/L) = -\lambda\frac{\partial T}{\partial Y}$ scaling

Scalings in DIS

- stochastic extension of Balitsky-Kovchegov equation

$$\frac{\partial T}{\partial Y} = \alpha_S(Q^2)\left[\chi(-\partial_L)T - T^2\right]$$

- α_S running
  - $\alpha_S \sim 1/logQ^2$
  - T^2 is expected to follow the scaling
  - but both $L\partial T/\partial Y$ and $\chi(-\partial_L)$ cannot follow the same scaling
  - two solutions:
    - Running Coupling I: $\tau = L - \lambda\sqrt{Y}$
    - Running Coupling II: $\tau = L - \lambda Y/L$ (see G. Beuf, arXiv:0803.2167)

Scalings in DIS

- extended Balitsky-Kovchegov equation

$$\frac{\partial T}{\partial Y} = \alpha_S \left[\chi(-\partial_L) T - T^2 + \sqrt{\alpha_S^2 \kappa T}\, \nu(L, Y) \right]$$

- α_S constant, ν
  - ν is a gaussian noise corresponding to the fluctuation of number of gluons
  - corresponds to pomeron loops (gluon splitting)
  - Diffusive Scaling: $(L - \lambda Y)/\sqrt{Y}$

Quality Factor Method

- test different scaling laws $\tau = \tau(Q^2, x; \lambda)$ on data $\sigma \sim F_2/Q^2$
- quality factor method
  - normalise data sets $v_i = \log(\sigma_i)$ and scaling laws $u_i = \tau_i(\lambda)$ between 0 and 1
  - order in u_i
  - define quality factor

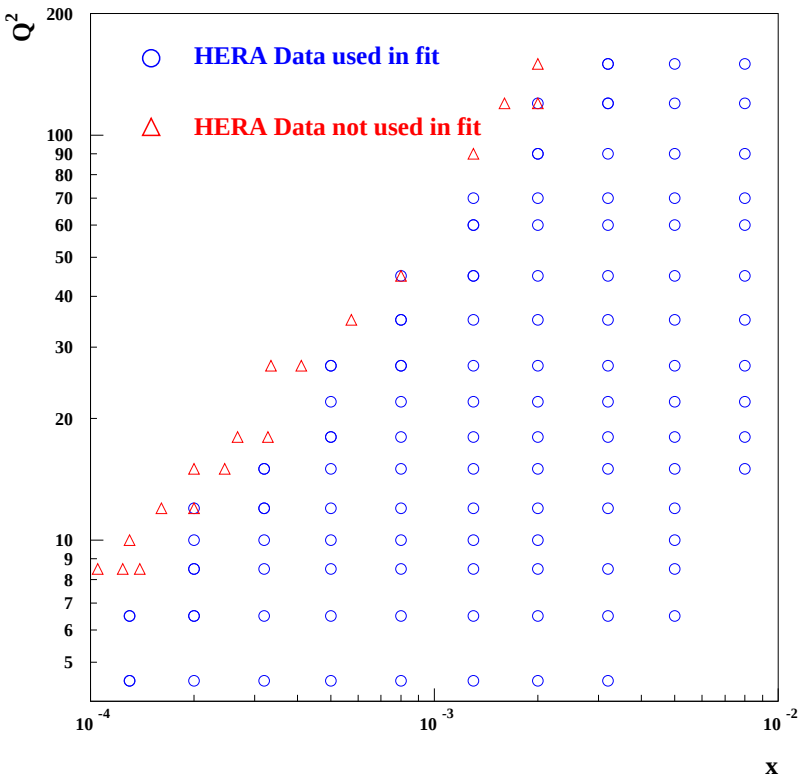
$$QF(\lambda) = \left[\sum_i \frac{(v_i - v_{i-1})^2}{(u_i - u_{i-1})^2 + \epsilon^2} \right]^{-1}$$

  - fit λ to maximase QF

- F. Gelis, R. Peschanski, L. Schoeffel, G. Soyez, hep-ph/0610435

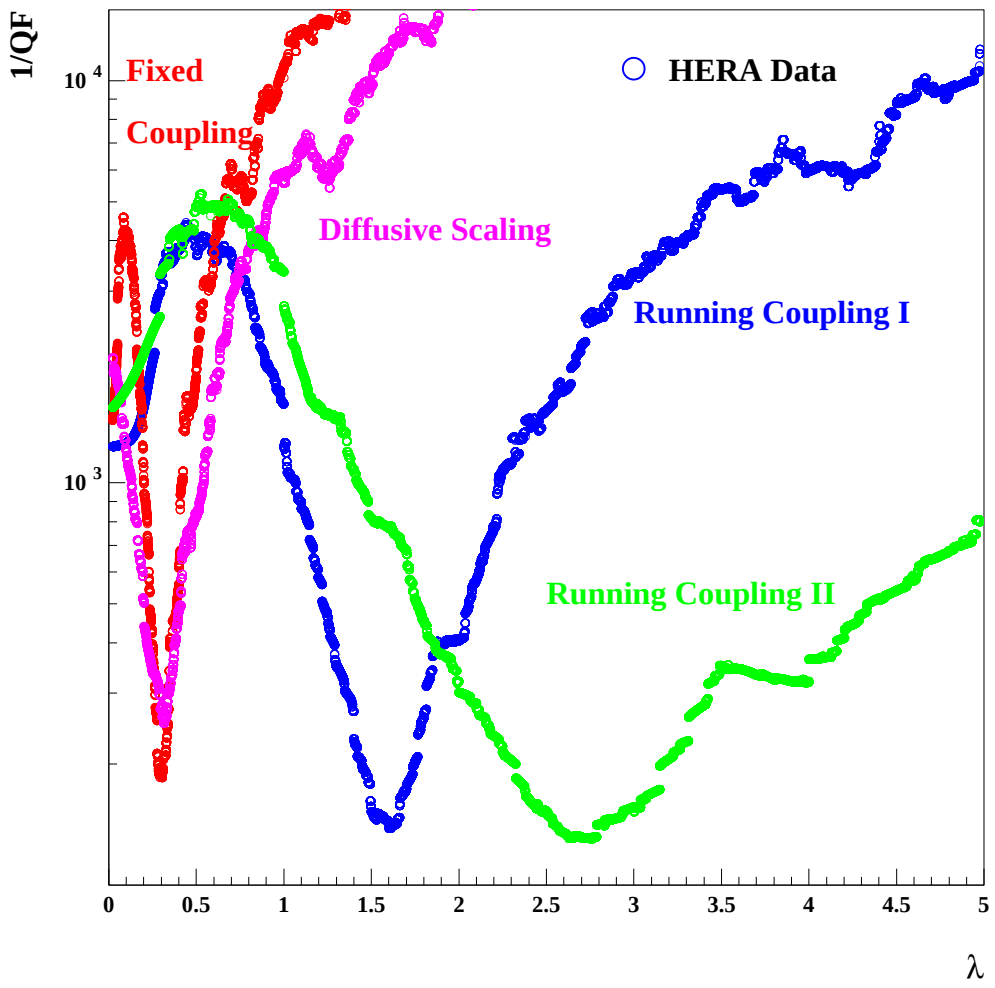
Scaling Tests in DIS

- combined F_2 measurements from H1/ZEUS
- stay in perturbative domain: $4 \leq Q^2 \leq 150\,\mathrm{GeV}^2$
- avoid region where valence quarks dominate: $x \leq 10^{-2}$
- avoid high y region where F_L contributes: $y \leq 0.6$
- 117 data points



Quality Factor

- comparison of $1/QF$ for FC, RCI, RCII, DS



Comparison of Different Scalings

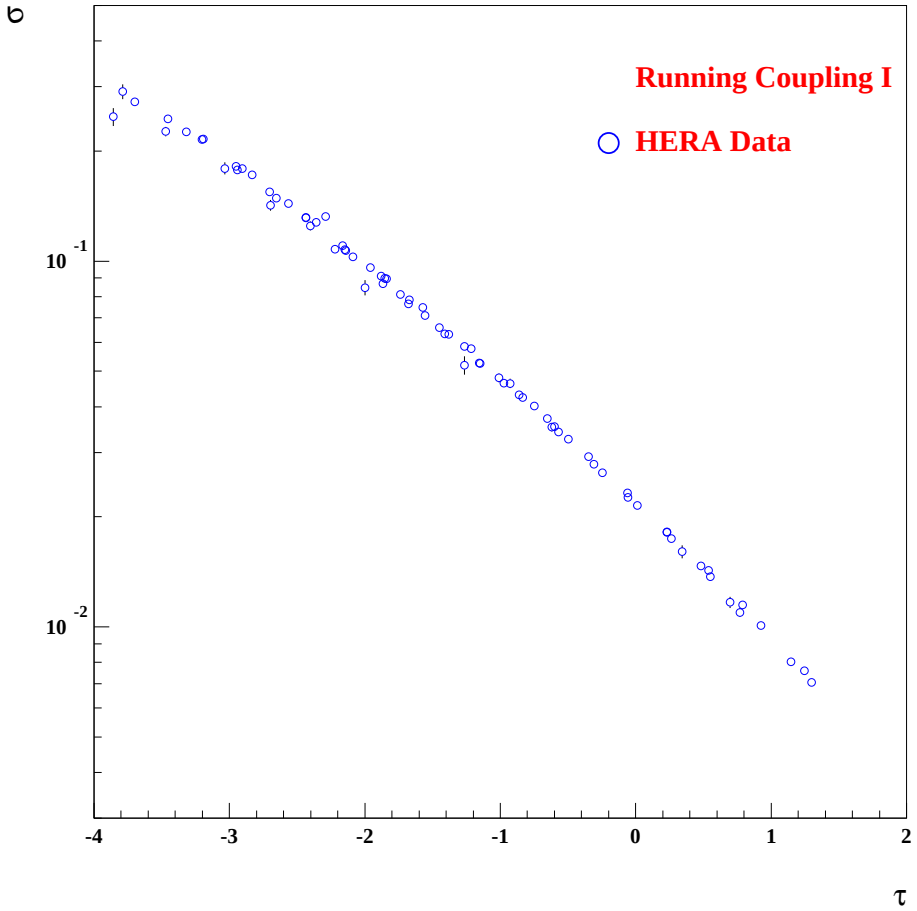
- value of parameters and QF for $Q^2 \geq 4$ GeV^2

scaling	λ	$1/QF$	
Fixed Coupling	0.31	150.2	$\tau = \log Q^2 - \lambda \log(\frac{1}{x})$
Running Coupling I	1.61	137.9	$\tau = \log Q^2 - \lambda \sqrt{\log(\frac{1}{x})}$
Running Coupling II	2.76	124.3	$\tau = \log(Q^2/0.2^2) - \lambda \frac{\log(\frac{1}{x})}{\log(Q^2/0.2^2)}$
Diffusive Scaling	0.31	210.7	$\tau = \frac{\log Q^2}{\sqrt{\log \frac{1}{x}}} - \lambda \log(\frac{1}{x})$

- FC, RCI and RCII favoured, DS disfavoured
- no significant improvement with additional parameteres (Q_0, Y_0)
- results compatible with our previous study using older data (arXiv:0803.2186)

Scaling Plot

- Running Coupling I: $\tau = \log Q^2/Q_0^2 - \lambda \log(\frac{1}{x})$
- dilute regime (no saturation) $\rightarrow \tau > 0$
- τ can be shifted by changing Q_0



Fits to HERA Data

- fit to HERA data inspired by RCI

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda\sqrt{\log(\frac{1}{x}) - Y_0}$$

$$\sigma = N\exp(-\alpha\tau)\exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)$$

- fit formula deduced from the dipole amplitude with saturation (Gregory Soyez) with asymptotic expression of the Airy function which is a solution of Balistsky-Kovchegov equation
- 6 fit parameters: λ, α, β, Q_0, Y_0, N
- explicit moderate scaling violation: $(\log 1/x - Y_0)^{1/4}$
  - fits performed with and without the scaling violation term (predicted by the dipole model)
- τ must be positive in the dilute regime

Fit Results

- fit variables:

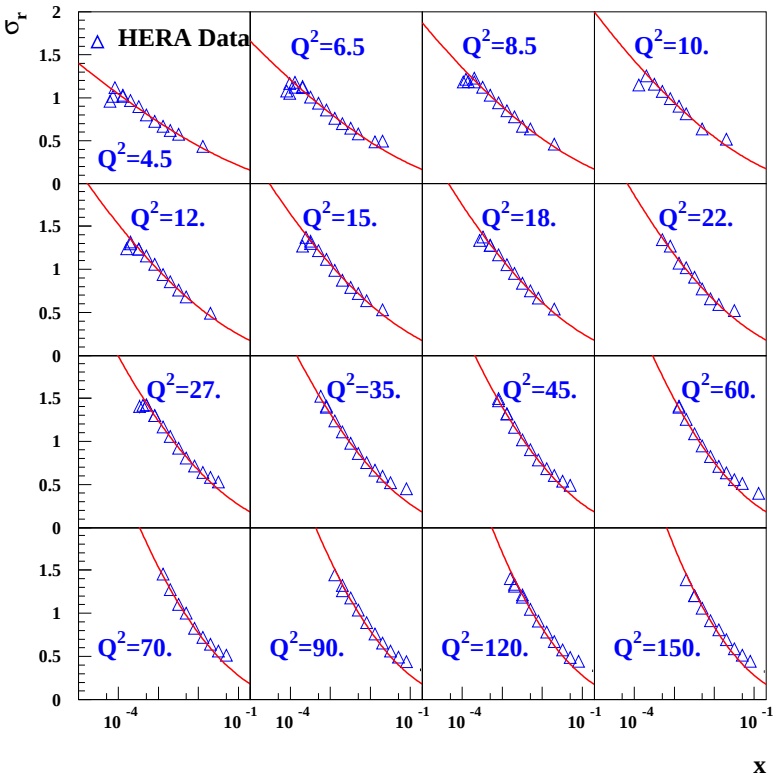
$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda\sqrt{\log(\frac{1}{x}) - Y_0}$$

$$\sigma = N\exp(-\alpha\tau)\exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)$$

- Fit I: $\chi^2 = 130.1$ for 117 points ($\chi^2/dof = 1.2$)
- Fit II (without the scaling violation term): $\chi^2 = 119.0$

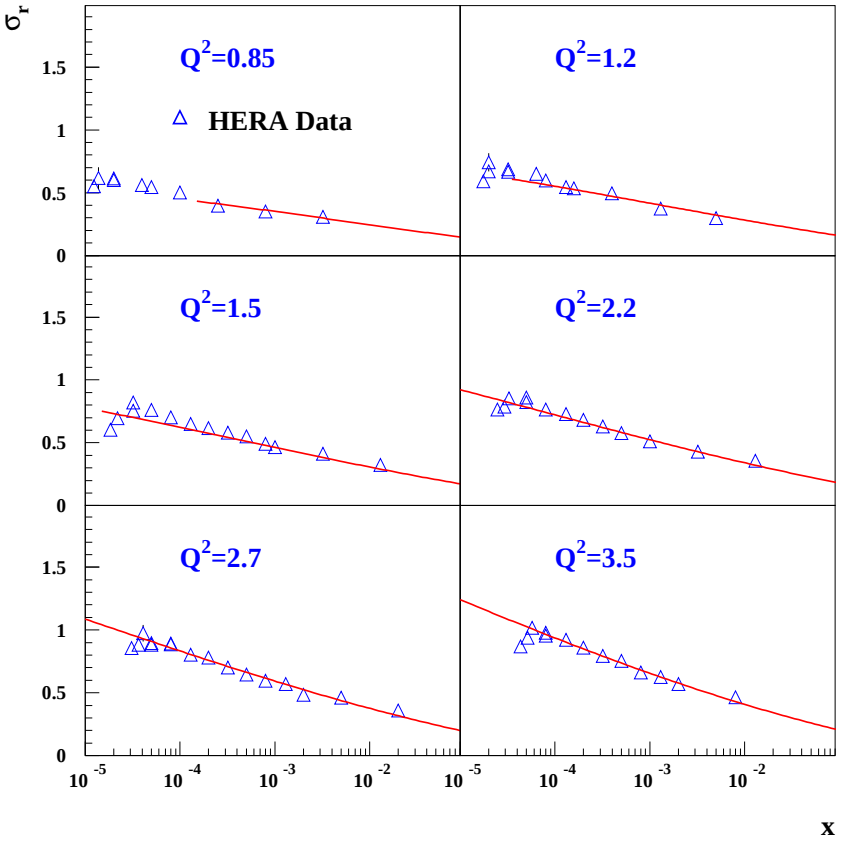
Parameter	Fit I	Fit II
λ	1.54 ± 0.02	1.54 ± 0.02
α	0.34 ± 0.01	0.18 ± 0.01
β	0.24 ± 0.01	0.18 ± 0.01
Q_0	0.079 ± 0.01	0.064 ± 0.01
Y_0	-1.46 $\pm$ 0.02	0.50 ± 0.02
N	$0.51 \pm 0{,}01$	$0.72 \pm 0{,}01$

Fit Results



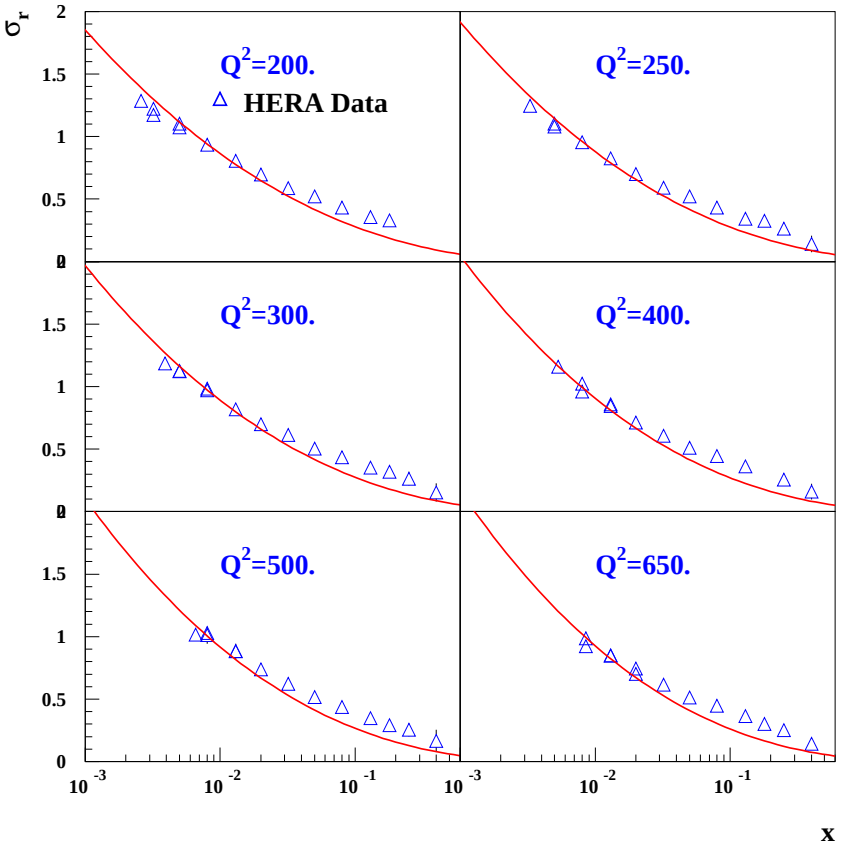
- good description of HERA data at low Q^2 and low x
- fit does not describe the turn-over of σ_r at low x (high y)
  $\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$
- model of F_L using RCI needed (in progress)

Extrapolation to Q^2



- fair description of data at lower Q^2
- parameterisation of F_L needed in order to describe high y data

Extrapolation to Q^2



- high x not well described
- valence quarks needed

Other Fits

- similar formula for RCII:

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda \frac{\log(\frac{1}{x}) - Y_0}{\log(\frac{Q^2}{Q_0^2})}$$

$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)$$

  - $\chi^2 = 190.4 \rightarrow$ worse than RCI
- similar formula for FC:

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda \log(\frac{1}{x})$$

$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^2}{(\log 1/x - Y_0)}\right)$$

  - $\chi^2 = 156.4 \rightarrow$ worse than RCI
  - $\chi^2 = 230.5$ without the scaling violation term

Conclusion

- different scalings studied in F_2 data: Fixed Coupling, Running Coupling I and II, Diffusive Scaling
- Fixed Coupling, Running Coupling I and II lead to a good description of data using the QF formalism
- Diffusive Scaling disfavoured

- fit of F_2 data using RCI (parameterised with or without moderate scaling violations) leads to a good description of data at low Q^2 and low x
- fits disfavour RCII and FC

- outlook:
  - fits of lower Q^2 data in the saturation region
  - fits of high y data including F_L parameterisation
  - comparision with numerical solution of BK equation with α_S running