UHE neutrinos: current non-conservation, mass scales, saturation

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by definition, UHE corresponds to

\[ E_\nu > 10^7 \text{ GeV} \]

A lot of interesting papers on

\[ \sigma^{\nu N}(E_\nu) \]

with \( E_\nu \) up to

\[ 10^{15} \text{ GeV} \]
Massless approximation and DLLA

\[ \sigma^{\nu N} = \frac{1}{s} \int_s^1 dQ^2 \int_{x_0}^1 \frac{dx}{x} \left( \frac{d\sigma}{dx dy} \right) \]

\[ \frac{d\sigma}{dx dy} = \frac{G_F^2 s}{2\pi} \left( \frac{m_W^2}{m_W^2 + Q^2} \right)^2 \left[ (1 - y)F_L + (1 - y + \frac{y^2}{2})F_T \right] \]

\[ \sigma^{\nu N} \propto \int dQ^2 \left( \frac{m_W^2}{m_W^2 + Q^2} \right)^2 \exp \sqrt{C \log(1/x) \log \log Q^2} \]

W-propagator introduces cut-off. Then the scale is

\[ Q^2 \sim m_W^2 \]
The estimate

\[ Q^2 \sim m_W^2 \]

is not unreasonable only for light flavors.

The top-bottom current needs special care. The phenomenon of Charged Current Non-Conservation (CCNC) pushes the scale up to

\[ \sim m_t^2 \]
to be discussed:

- Charged Current Non-Conservation (CCNC) effects in UHE neutrino interactions.
- Top-bottom current - new “hardness” scale
- Scales and saturation
$F_L$ as a carrier of CCNC effects

weak currents are not conserved..., but in what way?

For longitudinal/scalar W-boson in

$$W_L \rightarrow t\bar{b} \propto \varepsilon_{L}^{\mu} J_{\mu} \propto \partial_{\mu} J_{\mu} \propto m_t \pm m_b$$

Therefore,

$$F_L \propto \varepsilon_{L}^{\mu} T^{\mu\nu} \varepsilon_{L}^{\nu}$$

provides a measure of CCNC effects.
to calculate \( F_L = Q^2 \sigma_L / 4\pi^2 \alpha_W \)

one needs gauge invariant sum of diagrams like this

\( \mathcal{F} \) - un-integrated gluon density
\( \kappa \) - gluon momentum
\( z, k \) - Sudakov’s variables of t-quark

\[
\frac{d\sigma_L(x, Q^2)}{dzd^2k} = \frac{\alpha_W}{\pi} \int \frac{d^2\kappa}{\kappa^4} \alpha_S(q^2) \mathcal{F}(x, \kappa^2) (V_S + A_S + V_P + A_P)
\]

Two components: \( \bar{t}b \)-state with angular momentum \( L = 0 \) and \( L = 1 \): S-wave and P-wave.

- The appearance of P-wave - manifestation of CCNC
**S-wave:** \(|W\rangle \rightarrow |t\bar{b}, L = 0\rangle

\[
V_S(m_t, m_b) = \frac{1}{Q^2} \left\{ 2Q^2 z(1 - z) + (m_t - m_b) [(1 - z)m_t - zm_b] \right\}^2 \\
\quad \times \left( \frac{1}{k^2 + \varepsilon^2} - \frac{1}{(k - \kappa)^2 + \varepsilon^2} \right)^2
\]

\[
A_S(m_t, m_b) = \left( \frac{g_A}{g_V} \right)^2 V_S(m_t, -m_b)
\]

\[
\varepsilon^2 = z(1 - z)Q^2 + (1 - z)m_t^2 + zm_b^2
\]
**P-wave:** \(|W\rangle \rightarrow |t\bar{b}, L = 1\rangle\)

Scalar \(W\) → wrong helicities → suppression at \(Q^2 \gg m_q^2\)

\[
V_P(m_t, m_b) = \frac{1}{Q^2(m_t - m_b)^2} \left( \frac{k}{k^2 + \varepsilon^2} - \frac{k - \kappa}{(k - \kappa)^2 + \varepsilon^2} \right)^2
\]

\[
A_P(m_t, m_b) = \left( \frac{g_A}{g_V} \right)^2 V_P(m_t, -m_b)
\]

\[
\varepsilon^2 = z(1 - z)Q^2 + (1 - z)m_t^2 + zm_b^2
\]

This is higher twist but

\[
Q^2 < m_W^2 \ll m_t^2
\]
soft gluons, $\kappa^2 \lesssim k^2 + \varepsilon^2$, - DLLA

the DLLA (soft gluon) contribution to the P-wave or CCNC component of

$$F_L = F_L^S + F_L^P$$

is dominated by highly asymmetric configurations with

$$z \sim 1 - \frac{m_b^2}{m_t^2 + Q^2}.$$

Then

$$F_L^P(x, Q^2) \simeq \frac{m_t^2}{m_t^2 + Q^2} \int_{m_b^2}^{m_t^2} \frac{d\varepsilon^2}{\varepsilon^2} \frac{\alpha_S(\varepsilon^2)}{3\pi} G(x, \varepsilon^2)$$
The CCNC effect

\[ \propto \frac{m_t^2}{(m_t^2 + Q^2)} \]

survives in UHE neutrino interactions because the W-propagator allows only “small” \( Q^2 \)

\[ Q^2 < m_W^2 \ll m_t^2 \]

It is enhanced by the gluon density factor \( G(x, m_t^2) \).

This CCNC-term is not a property of the interaction of \( \bar{t}b \)-dipole with the target but the property of the light-cone density of \( \bar{t}b \)-states (Fiore & VRZ 2008).
To DLLA the CCNC contribution to $\sigma^{\nu N}$ is estimated as

$$\sigma^{\nu N}_{CCNC} \simeq 0.43 \times 10^{-31} \text{ cm}^2$$

for

$$E_\nu = 10^{12} \text{ cm}^2$$

(the gluon density $G(x, k^2)$ from Ivanov and Nikolaev 2003)

The frequently used massless approximation for different gluon densities (E. Henly and J. Jalilian-Marian, 2006) gives for $E_\nu = 10^{12} \text{ GeV}$

$$0.2 \times 10^{-31} < \sigma^{\nu N} < 1.5 \times 10^{-31} \text{ cm}^2$$
In massless approximation (CCNC neglected) unitarity corrections to $\sigma^{\nu N}$ found to be 50 per cent effect (Kutak and Kwiecinski 2003): unitarity turns

$$\sigma_{CC}^{\nu N} \simeq 1. \times 10^{-31} \text{cm}^2$$

at $E_\nu = 10^{12}$ GeV into

$$\sigma_{CC}^{\nu N} \simeq 0.5 \times 10^{-31} \text{cm}^2$$
scales and saturation

Saturation depends on scale. First higher twist correction

\[ \sim \frac{\alpha_s(Q^2)}{Q^2} \frac{G(x, Q^2)}{\pi R^2} \]

The CCNC hardness scale \( m_t^2 \) is much “harder”: than the mass scale \( Q_0^2 \) for light flavors:

\[ m_t^2 \gg Q_0^2 \lesssim m_W^2 \]

the ratio of saturation corrections is apparently very small

\[ \frac{\delta \sigma(CCN C)}{\delta \sigma(massless)} \sim \frac{Q_0^2}{m_t^2} \frac{\alpha_s(m_t^2) G(x, m_t^2)}{\alpha_s(Q_0^2) G(x, Q_0^2)} \ll 1 \]
Saturation affects strongly the light quark contribution to $\sigma^{\nu N}$ but leaves the top-bottom CCNC term intact.
Conclusions

The lower (DLLA) estimate for the CCNC contribution to $\sigma^{\nu N}$ presented for UHE neutrinos.

Found that

$$\sigma^{\nu N}_{CCNC} \sim \sigma^{\nu N}(massless)$$

Additional momentum to the CCNC effect in its competition with massless calculations gives the unitarity suppression which is much stronger for massless cross sections.